A Permutation Version of the Durbin-Watson Test for Serial Correlation.

Eugene Canjels, New School University[1]

October, 2002.

Abstract

To test for serial correlation in regression errors, this paper proposes a new permutation test based on the Durbin-Watson statistic. This test is asymptotically correctly-sized for error distributions with finite variance. A Monte Carlo study shows that, for distributions with or without finite variance and where $n \geq 20$, the new permutation test controls size better than the Durbin-Watson test. The new permutation test achieves this higher control at a very low cost: when the Durbin-Watson test is correctly sized, power losses are usually undetectable, and in no case do they exceed 1.5%. Moreover, the new permutation test dominates a seemingly similar bootstrap procedure; in particular, the new test performs significantly better in the critical case of heavy-tailed distributions. *JEL classification*: C12; C14; C22. *Keywords*: Durbin-Watson test; Serial correlation; Permutation tests.

[1] Department of Economics, Graduate Faculty, New School University, 80 Fifth Avenue, 5th floor, New York, NY 10011. Tel: (212) 229-5901. Fax: (212) 229-5903. Email: <canjelse@newschool.edu>.

1. Introduction

The Durbin-Watson (DW) test continues to be one of the most common procedures for testing serial correlation in regression errors. One constraint of the DW test is its assumption that the regression errors are normally distributed. To address this limitation, others have proposed alternative tests (including a variety of non-parametric tests) that relax the normality assumption. However, many researchers currently view the DW as the test of choice because of (i) its high power; and (ii) its limited size distortions. Nevertheless, some size distortions exist for error distributions that are highly skewed and/or heavy-tailed.[2]

Of particular interest for the purposes of this paper is Schmoyer's research (1994), in which he develops a permutation test based on ideas from Fisher (1935) and Pitman (1937). Permutation procedures are attractive since they are usually exact (in the sense that they do not rely on asymptotic approximations) and as powerful asymptotically as their parametric counterparts (Hoeffding, 1952). However, this exactness is lost when permutation tests are applied to regression residuals, rather than to regression errors. Schmoyer's test, as indicated by the simulations described in his paper, shows large size distortions for finite samples, thus casting doubt on the usefulness of his test for applied research.

To overcome the described limitations of both DW and Schmoyer, this paper proposes a permutation test — henceforth the Permutation Durbin-Watson (PDW) test — that is based on the original DW statistic. The PDW relaxes some of the technical assumptions of Schmoyer. This paper then shows that, without imposing any restrictions on the design matrix, the PDW based on ordinary least-squares (OLS) residuals is asymptotically valid for errors with only second moments. Simulations show that the PDW test performs almost as well as the DW for normal distributions and is clearly su-

[2] See King (1987) for an excellent review of this literature. For studies of the robustness of the DW test, see section 3 of this paper and the references therein.

perior for heavy-tailed distributions. This establishes the PDW test as a widely applicable and serious alternative to the DW.

The remainder of this paper is structured as follows. Section 2 first introduces the basic idea of a permutation test and illustrates the difficulties of designing such a test in the context of a linear regression. This section then presents the PDW test and proves its asymptotic validity. Since the PDW appears to be similar to a bootstrap test that is based on the DW statistic, we discuss that situation in some detail. Section 3 then presents the results of a Monte Carlo study that, for finite samples, compares the size and power properties of the DW, PDW and bootstrap tests.

2. A permutation test for serial correlation

2.1. The Durbin-Watson test

Consider the linear regression model

$$y = X\beta + e \tag{1}$$

where $y = (y_1, ..., y_T)'$, $e = (e_1, ..., e_T)'$, $X = (x_1', ..., x_T')'$ is a $T \times k$ matrix of full rank, and $\beta = (\beta_1, ..., \beta_k)'$ is a k-vector of unknown parameters. The objective is to test the hypothesis that the e_t's are serially uncorrelated. The Durbin-Watson test is designed to test the hypothesis $\phi = 0$ in the equation $e_t = \phi e_{t-1} + \eta_t$, where the η_t's are normally, independently, and identically distributed. Let $M = I - X\,(X'X)^{-1}\,X'$; then, the OLS residuals of (1) are given by $r = Me$. The Durbin-Watson statistic is defined as

$$d = \frac{\sum_{t=2}^{T}(r_t - r_{t-1})^2}{\sum_{t=1}^{T} r_t^2} = \frac{r'Wr}{r'r} = \frac{e'MWMe}{e'Me} \tag{2}$$

where

$$W = \begin{bmatrix} 1 & -1 & 0 & . & . & . & . & 0 \\ -1 & 2 & -1 & & & & & 0 \\ 0 & -1 & 2 & & & & & 0 \\ . & & & . & & & & . \\ . & & & & . & & & . \\ . & & & & & . & & . \\ . & & & & & & 2 & -1 \\ 0 & 0 & 0 & . & . & . & -1 & 1 \end{bmatrix} \tag{3}$$

Although its distribution depends on the design matrix X, the statistic d is pivotal under the null hypothesis and the assumption of normality. Imhof (1961) describes methods that enable this distribution to be simulated, and recent advances in computer technology have also made this simulation feasible for applied work. Hereinafter, we will use the term "Durbin-Watson test" (or "DW test") to describe specifically that test based on the statistic d and which uses critical values based on the assumption of normality. In particular, the term "DW test" does not refer to the original bounds test as proposed in Durbin and Watson (1950, 1951).

If the distribution of the e_t's is unknown, then the exact distribution of d will also be unknown. However, if the parameters β are consistently estimated and the errors have second moments, the statistic $\frac{1}{2}\sqrt{n}\,(d-2)$ will be asymptotically distributed as a standard normal.[3] Since this property is also true when the errors are normally distributed, the DW test is an asymptotically valid test (i.e., the critical values are asymptotically correct).

2.2. Permutation test using the true errors and the DW statistic

If the e_t's were observable and iid under the null hypothesis of no serial correlation, then one can use the DW statistic to perform a permutation test with an exact size. To illustrate, let Z be a random permutation matrix of size $T \times T$. The DW statistic based on errors, rather than on residuals, is:

$$d_e = \frac{\sum_{t=2}^{T} (e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2} \tag{4}$$

If we perform a random permutation, $\tilde{e} = Ze$, the DW statistic based on the permutated errors is

[3] For instance, Phillips and Loretan (1991).

given by

$$\tilde{d}_e = \frac{\sum_{t=2}^{T}\left(\tilde{e}_t - \tilde{e}_{t-1}\right)^2}{\sum_{t=1}^{T}\tilde{e}_t^2} \tag{5}$$

Since the errors are exchangeable under the null hypothesis, then for any permutation matrix Z, $\tilde{d}_e$ has the same distribution as d_e. Of course, the distributions of d_e and $\tilde{d}_e$ will be different under the alternative hypothesis, and it is precisely this characteristic that permutation tests exploit.

Consider the set of all permutations and let D be the set of corresponding values of $\tilde{d}_e$, including the observed d_e. The set D contains $n!$ elements. Since the e_t's are exchangeable, the probability that d_e has a rank ρ in the set D is equal to $\frac{1}{n!}$. Correspondingly, the probability that d_e has a rank smaller or equal to ρ is equal to $\frac{\rho}{n!}$. Therefore, to construct a test with an exact size of α, we reject the null hypothesis when $\rho \leq \alpha \cdot n!$.[4] Alternatively, when we observe a rank ρ, we can calculate an exact p-value as $\frac{\rho}{n!}$.

When $n!$ is prohibitively large, we can draw a random sample of size m from the set D. Next, we expand this with the statistic d_e, and call this set E. Then, the probability that d_e has a rank smaller or equal to ρ in E is $\frac{\rho}{m+1}$, and we can construct a test of size α by rejecting the null hypothesis when $\rho \leq (m+1)\,\alpha$, or we can present the p-value $\frac{\rho}{m+1}$.[5] These p-values are exact even for small m, although the power of the test will increase as the sample size m increases. One typically chooses m such that $(m+1)\,\alpha$ is an integer or, when presenting p-values, such that $m+1$ is a round number.[6]

2.3. Resampling residuals

The application of permutation tests to linear regressions is hampered by the fact that OLS residuals are not exchangeable unless X contains only a vector of constants. In this case, with $\tilde{r} = Zr$, the DW

[4] The test is only of size α if α is chosen such that $\alpha \cdot n!$ is an integer. Otherwise, the test has level α but size $[\alpha \cdot n!]\,/n!$, where $[\alpha \cdot n!]$ indicates the largest integer less than or equal to $\alpha \cdot n!$.

[5] Again, assuming that $(m+1)\,\alpha$ is an integer. See previous footnote.

[6] The sets D and E should properly be referred to as multisets, since some of the values may be repeated. If this is the case, the rank of d_e is not necessarily uniquely defined. The argument presented is correct provided that the ranking of identical elements in the multisets D or E is random. In many cases, with continuous random variables and proper test statistics, the probability that two elements are identical is negligible, and can be ignored for practical purposes.

statistic calculated from permutated residuals:

$$\frac{\sum_{t=2}^{T}(\tilde{r}_t - \tilde{r}_{t-1})^2}{\sum_{t=1}^{T}\tilde{r}_t^2} = \frac{r'Z'WZr}{r'Z'Zr} = \frac{e'MZ'WZMe}{e'Me} \tag{6}$$

does not have the same distribution as the DW statistic:

$$d = \frac{\sum_{t=2}^{T}(r_t - r_{t-1})^2}{\sum_{t=1}^{T}r_t^2} = \frac{r'Wr}{r'r} = \frac{e'MWMe}{e'Me} \tag{7}$$

Since, as n becomes large, the residuals converge to the true errors, one might expect this procedure to work asymptotically. The theoretical results in the appendix do, in fact, confirm this expectation, but subject to the caveat that the approximations will not typically be precise enough to be of use in finite samples.[7]

Since

$$d = \frac{e'MWMe}{e'Me} = \frac{r'MWMr}{r'Mr} \tag{8}$$

we might also use permutations of the residuals in the latter formula. This leads to:

$$\tilde{d} = \frac{r'Z'MWMZr}{r'Z'MZr} \tag{9}$$

The permutation procedure using (9) performs very well in small samples. It is this procedure that we refer to as the Permutation Durbin-Watson (PDW) test.

Note that if we define

$$\tilde{y} = X\hat{\beta} + \tilde{r} \tag{10}$$

then $\tilde{d}$ is the DW statistic from the regression of $\tilde{y}$ on X. This immediately makes clear that the PDW test is similar to a bootstrap procedure (henceforth referred to as the bootstrap DW or BDW) where we resample the residuals with replacement and approximate the distribution of the test statistic under the null hypothesis by using these resampled residuals.[8] Since the population of the resampling procedure in the BDW satisfies the DW test conditions for asymptotic normality, the bootstrap distribution will

[7] Schmoyer's (1994) permutation procedure is closely related to this method. Simulations described in his own paper also demonstrate poor performance.

[8] This procedure is almost identical to the procedure suggested by De Beer and Swanepoel (1989). The only difference is that they make the bootstrap sample size dependent on the data instead of setting it equal to the original sample size T.

also be asymptotically normal with the same mean and variance as the DW statistic. Therefore, we can use the bootstrap distribution to obtain asymptotically valid critical values or p-values. (The asymptotic theory of the PDW test is more complicated and will be presented in the next section, 2.4.)

The question arises as to why one should analyze the PDW when an apparently similar procedure is readily available. To understand the motivation for this analysis, note that the BDW will typically start to function poorly when errors have distributions with heavy tails. Yet, this is precisely when the DW test behaves poorly. In contrast, permutation tests are often robust against heavy tails. For instance, when the design matrix X consists only of a constant, the PDW test is exact for every error distribution and sample size. Section 3 then investigates whether this result extends to the case where the design matrix is less trivial.

2.4. Asymptotic theory

We begin with a general discussion of the asymptotic justification for using OLS residuals in permutation tests. We then apply the techniques described in Theorems 1 and 2 below to the PDW test. Assume that e is a vector of iid random variables with $E(e) = 0$ and $E\left(e^2\right) = \sigma^2 < \infty$. Let $\theta_T(e)$ be a test statistic that is calculated from the true errors of the regression by using T observations. Depending on the test statistic, we reject the null hypothesis when $\theta_T(e)$ is 'small' (as in the case of the DW statistic and the alternative of positive serial correlation), or 'large' (when the alternative is negative serial correlation). Next, we assume that we reject the null hypothesis when $\theta_T(e)$ is small. We draw m independent random permutation matrices Z_j, for $j = 1, ..., m$. Let $\theta_T(Z_j e)$ be the corresponding test statistics. Then, the statistic $\sum_{j=1}^{m} I\{\theta_T(Z_j e) \leq \theta_T(e)\} + 1$ gives the rank of $\theta_T(e)$.[9] The statistic $p_T(e) = (m+1)^{-1}\left(\sum_{j=1}^{m} I\{\theta_T(Z_j e) \leq \theta_T(e)\} + 1\right)$, gives the

[9] For convenience, we ignore the possibility of ties. We can account for this straightforwardly by randomizing the ordering in the case of ties (as explained in footnote 6), albeit with the cost of notational complexity.

corresponding p-value. That is,

$$P\left(p_T\left(e\right) \leq \alpha\right) = \alpha. \tag{11}$$

The permutation test based on residuals rather than on errors provides asymptotically correct p-values if $p_T\left(r\right) - p_T\left(e\right) \xrightarrow{p} 0$. The following theorem provides fairly general sufficient conditions to establish asymptotic validity.

Theorem 1 *Suppose that the following three conditions are satisfied*

1. $\theta_T\left(r\right) = \theta_T\left(e\right)$
2. $\lim_{T\to\infty} P\left\{\theta_T\left(Zr\right) \leq \theta_T\left(e\right) \leq \theta_T\left(Ze\right)\right\} = 0$
3. $\lim_{T\to\infty} P\left\{\theta_T\left(Ze\right) \leq \theta_T\left(e\right) \leq \theta_T\left(Zr\right)\right\} = 0$

Then $p_T\left(r\right) - p_T\left(e\right) \xrightarrow{p} 0$.

All formal proofs are in the appendix. The intuition, however, is straightforward: if the probability that the $\theta_T\left(e\right)$ falls between $\theta_T\left(Zr\right)$ and $\theta_T\left(Ze\right)$ approaches zero as T approaches infinity, then $T_T\left(Zr\right)$ can be used in the permutation test instead of $T_T\left(Ze\right)$.

The following theorem suggests how one can verify the conditions in Theorem 1.

Theorem 2 *Suppose that the following two conditions are satisfied*

1. $g\left(T\right)\left(\theta_T\left(Ze\right) - \theta_T\left(Zr\right)\right) \xrightarrow{p} 0$
2. $\lim_{\varepsilon\to 0} \lim_{T\to\infty} P\left\{\left|g\left(T\right)\left(\theta_T\left(e\right) - \theta_T\left(Ze\right)\right)\right| < \varepsilon\right\} = 0$

Then $\lim_{T\to\infty} P\left\{\theta_T\left(Zr\right) \leq \theta_T\left(e\right) \leq \theta_T\left(Ze\right)\right\} = 0$ *and* $\lim_{T\to\infty} P\left\{\theta_T\left(Ze\right) \leq \theta_T\left(e\right) \leq \theta_T\left(Zr\right)\right\} = 0$.

The first condition can be trivially satisfied by letting $g\left(T\right)$ rapidly approach zero. The second condition imposes a restriction on how fast $g\left(T\right)$ can approach zero. For instance, with the DW statistic, one can easily show that one satisfies the first condition when $g\left(T\right) = \frac{1}{T}$. However, this violates the second condition, since $\frac{1}{T}\theta\left(e\right) \xrightarrow{p} 2$ and, by exchangeability, so does $\frac{1}{T}\theta\left(Ze\right)$. As the proof of Theorem 3 demonstrates, setting $g\left(T\right) = \sqrt{T}$ produces the appropriate scaling for the DW statistic, thus satisfying both conditions.

Now consider the PDW test:

Theorem 3 *Let* $\theta_T(e) = \frac{e^{\theta} MWMe}{e^{\theta} Me}$. *Then* $p_T(r) - p_T(e) \xrightarrow{p} 0$.

We require only the existence of second moments for e, which is a precondition identical to the assumptions made for the DW and the BDW tests. Perhaps most surprising is that we require no restrictions on the design matrix X, except full column rank for every sample size (and even this can be relaxed by letting $M = I - XX^*$, where X^* is the generalized inverse of X). For instance, there is no need to consistently estimate the coefficients in the regression. This relaxes the assumptions of the DW and BDW which, according to the current literature, do require consistent estimation of the regression coefficients. In the case where X consists only of a vector of constants, the PDW test is exact and requires no asymptotic justification, since the residuals are exchangeable.

3. Size and power in finite samples

To analyze the effects of size and power in finite samples, we perform a Monte Carlo experiment. As usual, we can reduce the dimensions of the experiment by taking into account invariance principles present in the problem. First, the DW, PDW, and BDW tests are invariant with respect to β and to σ^2. Second, these tests are also invariant to linear transformations of the data as X only enters their definition through the projection matrix $P = X(X'X)^{-1} X'$.

We can obtain guidance on choosing error distributions and design matrices by examining several previous studies on the robustness of the DW test. In particular, we examined Evans (1992), Ali and Sharma (1993), Bartels and Goodhew (1981), Eps and Eps (1977), Gastwirth and Selwyn (1980), Harrison and McCabe (1975) and the review article by King (1987).

The DW test is exact for normally distributed errors, and these studies show that it is also expected to work well for error distributions that have limited degrees of skewness and kurtosis. In contrast, certain distributions — some that are highly skewed or heavy-tailed — are highly problematic for the DW test. Interestingly, these researchers found that the DW test can be both conservative and liberal;

they conclude that the largest size problems occur at the 1% confidence level, while the 5% and 10% confidence levels are fairly robust.

We then consider frequently used non-normal distributions such as the uniform, central χ^2_1 and log-normal; and also distributions from two families that let one easily control the existence of moments. The first such type of distribution is the (Generalized) Tukey Lambda. This family of distributions was introduced in Hastings e.a. (1947) and has many properties that make it convenient for Monte Carlo studies. There are various representations of the distribution; we use the one in Freimer e.a. (1988). The random variable X has a standardized Tukey Lambda distribution if

$$X = \frac{U^{\lambda_1} - 1}{\lambda_1} - \frac{(1-U)^{\lambda_2} - 1}{\lambda_2} \tag{12}$$

where the random variable U is uniform between 0 and 1, and is defined for every $(\lambda_1, \lambda_2) \in \Re^2$. For our purposes, two important characteristics are that the k-th moment exists if and only if $\min(\lambda_1, \lambda_2) > -k^{-1}$, and that the distribution is symmetric for $\lambda_1 = \lambda_2$.

The second family is that of (Paretian) stable distributions (see, e.g., Feller, 1971). For these distributions, moments exist up to the parameter α, which is called the index of stability. The skewness parameter is given by $\beta \in [-1, 1]$ with symmetry for $\beta = 0$. The Cauchy ($\alpha = 1$, $\beta = 0$)and normal ($\alpha = 2$) distributions are two special cases of the stable distribution.

We consider the following error distributions: normal, uniform, lognormal, centered chi-square with one degree of freedom, stable distributions with $\alpha \in \{0.4, 0.7, 1.0, 1.3, 1.6, 1.9\}$ and $\beta \in \{0, 1\}$, and Tukey Lambda distributions with $(\lambda_1, \lambda_2) \in \{(.5, .5), (2, .5), (1.5, 1.5), (2.5, 1.5), (2.5, 2.5), (-1/3, -1/3), (-2/3, -2/3), (-1, -1), (-1.3, -1.3)\}$. The first five Tukey Lambda distributions were chosen from the five classes described in Freimer e.a. (1988). The last four were chosen based on the existence of third, second and first moments. To conserve space, we only report in the table below the results for the: (i) normal; (ii) uniform; (iii) lognormal; (iv) Cauchy; (v) symmetric stable with $\alpha = 1.6$; (vi) Tukey Lambda with $\lambda_1 = \lambda_2 = -1/3$; and (vii) Tukey Lambda with $\lambda_1 = \lambda_2 = -2/3$.

(Complete tables are available from the author upon request.) We note that the distributions (iv), (v), and (vii) do not have second moments; therefore, none of these three tests is asymptotically justified.

The prior literature disagrees on the importance of the design matrix. For instance, Ali and Sharma (1993) show that, under the null hypothesis, the first four central moments of the DW test are not affected by the design matrix up to an order of $O\left(T^{-3}\right)$. However, Knight (1983) found that robustness sometimes depends critically on the design matrix. For the design matrices X, we present the following cases: (a) X consists of only a constant; (b) $x_t = (1, x_t^1)$ where x_t^1 is niid(0,1); (c) $x_t = (1, t)$, (d) $x_t = \left(1, t, x_t^1\right)$ where $x_t^1 = x_{t-1}^1 + \eta_t$ and η_t is iid uniform. We also considered, but again to save space chose not to present, the following designs:[10] $x_t = (1, x_t^1)$ where x_t^1 is stable with $\alpha = 1$ and $\beta = 1$; $x_t = \left(1, t, x_t^1\right)$ where $x_t^1 = .5x_{t-1}^1 + \eta_t$ and η_t is iid uniform; and $x_t = \left(1, t, x_t^1\right)$ where $x_t^1 = x_{t-1}^1 + \eta_t$ and η_t is iid stable with $\alpha = \beta = 1$.

We considered sample sizes of $T = 12, 20, 50$, and 100. To maximize the ability to objectively compare experiments — that is, across combinations of the design matrix, error distribution, and sample size — we treat any two experiments that have the same number of observations and the same data-generating process for the design matrix as having identical realizations of that design matrix. Across all the tests, the realized errors are identical for the same experiment. However, to avoid the occurrence of patterns due to statistical randomness, neither the errors nor the permutations are identical across experiments.

Some technical details on the experiments follow. We performed all simulations using GAUSS 3.2.32, which creates random variables with normal and uniform distributions by using a linear congruential uniform random number generator. These random variables are subsequently used to generate random variables with the lognormal, centered chi-square and the Tukey Lambda distributions. We use the methods by Chambers, Mallows and Stuck (1976) (coded in GAUSS by J. Huston McCul-

[10] Again, tables are available upon request.

loch and contained in the file “stab_rng.src”, which is available at the GAUSS Source Code Archive at American University), to generate the stable and Cauchy random variables. For the DW test, we calculate the p-values using Imhof’s (1961) procedure (as proposed by Koerts and Abrahamse (1969) and as implemented in GAUSS by Anurag Banerjee in the file “anurag.prc”, also available at the GAUSS Source Code Archive).

In the exercises investigating power, we construct what we call the Exact Durbin-Watson (EDW) test. This test uses the DW statistic, but it computes critical values by using the true distribution of the errors. The computation uses 100,000 simulations, and this process produces negligible distortions when applied to the DW test under normally distributed errors.

For the PDW test, we use 199 permutations. While this is usually enough to reach asymptotically maximal power, a larger number of permutations does lead to some power gains for 1% level tests against certain alternatives. We discuss this further in Section 3.3 below. While this determination about power suggests that one should choose a higher number of permutations when feasible in applied work, we were constrained by limited computer resources from exercising this choice.

For the BDW, we also used 199 samples to simulate the distribution under the null hypothesis. For some cases discussed below, we again increased that number in order to examine whether we could improve the performance of the BDW.

When we used a small sample size ($T = 12$), the problem of ties discussed in footnote 6 occasionally mattered; in this circumstance, we broke ties by using random rankings. All the experiments used 10000 simulations, which led to the following results for the DW test: standard errors of 0.00500, 0.00300, 0.00218, and 0.00099 on the 50th, the 10th, the 5th, and the 1st percentiles, respectively. We also note that, because the PDW and BDW tests involve additional uncertainties, these could cause the standard errors to be slightly higher.

3.1. Size

Table 1 reports the small sample (simulated) probabilities that a p-value falls below 0.10, 0.05, and 0.01, respectively, under the null hypothesis of no serial correlation. A bold and italic font indicates cases where the frequency differs from 0.10, 0.05, and 0.01 by more than 0.006, 0.00436, and 0.00198 respectively, each of which represents twice the standard error of the simulations. This table demonstrates a number of clear patterns that arise. To examine these, we first discuss the size distortions of the DW test; then, we compare the PDW test with the DW and the BDW tests. We conclude with a more detailed discussion about the size distortions of the PDW test.

Consistent with the findings described in other literature, the DW test demonstrates size problems for error distributions that have heavy tails and skewness; moreover, for these sample sizes, the performance does not improve as sample size increases. That is, size distortions are often equal or larger at $T = 100$ than at $T = 20$, even for those distributions that do have second moments. An interesting pattern that emerges is that, in large samples, the size distortions depend mostly on the error distribution and are almost independent of the design matrix. This is consistent with the theoretical findings of Ali and Sharma (1993) which are that, up to $O\left(T^{-3}\right)$, the first four central moments of the effect of non-normality on the DW test are free of regressors. For smaller samples, however, the design matrix can sometime play an important role. This finding provides a possible resolution to the disagreement about the importance of the design matrix.

A second interesting observation is that different sample sizes can show qualitatively different results. For instance, with samples of $T = 50$ and $T = 100$, we observe that when the error distribution causes size distortions, the 10% level is conservative, the 1% level is liberal and the 5% level can be liberal, conservative, or even approximately correct. On the other hand, for $T = 20$, the 5% level test is usually the most distorted and liberal, while the 1% and 10% level tests are close to being correctly-sized. Apparently, the conclusions of prior investigations regarding the correct level for 5%

and 10% tests must be qualified in that these levels are correct only in the case of larger samples.

PDW vs. DW: There is a simple, but important, conclusion that we can draw from comparing the PDW against the DW test: the PDW test controls size better than the DW test for almost every error distribution and design matrix when $T \geq 20$. When $T = 12$, the same conclusion applies, but only for designs (a) and (b) and not for designs (c) and (d). (Note: all design labels are so indicated in the table in the appendix and also in the earlier part of Section 3 of this paper.)

PDW vs. BDW: While the BDW does control size better than the DW in the specific case of heavy-tailed distributions, the PDW test controls size better than the BDW test for almost every error distribution and design matrix when $T \geq 20$. When $T = 12$, the following results apply. In design (a), the PDW dominates the BDW. In designs (b) and (d), the BDW dominates the PDW. In design (c), the PDW and BDW are about equal. When $T \geq 50$, the PDW closely approximates the correct size for all error distributions with second moments. However, the BDW still has some size distortions at $T = 100$ for the lognormal and the Tukey Lambda (vi) distributions.

For error distributions without second moments, the PDW is not always completely successful. In particular, the Cauchy distribution shows some size distortions even at $T = 100$. For other heavy-tailed distributions, some size distortions exist at smaller sample sizes. These results suggest that the assumption of second moments of the error distribution is too strict and can be substantially relaxed.

Obviously, since the PDW is exact only when the design matrix consists of a vector of constants, the size distortions will depend on the design matrix. We have not undertaken a full investigation into those characteristics of the design matrix that might affect the performance of the PDW tests. Nevertheless, the evidence thus far suggests that the design matrix is fairly important when there are small samples and heavy-tailed distributions.

3.2. Power

To investigate the power properties of the DW, the PDW and the BDW, we use 100 startup observations and consider three AR(1) error processes with coefficients 0.1, 0.5, and 1.0, which respectively cover small, moderate, and high serial correlation. We then compare the power of these tests to the power of the EDW test, which uses critical values simulated from the true error distribution. Table 2 reports the results. To assist the reader in identifying the most interesting findings, we have used bold- and italic-face to note power differences larger than 1% (in absolute value).

The power of the EDW test is mostly a function of the sample size and the serial correlation coefficient. The design is relatively unimportant; it plays some role only when there are moderate serial correlations and very small sample sizes. The error distribution is typically significant only when the distributions have heavy tails.

Table 2 demonstrates that several interesting patterns arise. Let us examine first the comparison of the DW test to the EDW test. It appears that the DW test is sometimes more powerful than the EDW test. However, Table 1 demonstrates that this is only the case when the DW test is liberal; the apparent power gains are a consequence of not controlling the size of the test. On the other hand, the power of the DW test is occasionally much lower; this is due to the conservatism of the DW test. (The largest loss observed is 50%.) This suggests that one can obtain significant power gains by better controlling the level of the test.

Second, for varying degrees of serial correlation, we compare the PDW with the infeasible EDW and the feasible DW. The results for the four different design matrices closely mimic each other. Because the PDW is exact for the first design, we limit our discussion to this case and leave it for the reader to verify that our conclusions are valid independent of the design.

Low serial correlation: When there are small degrees of serial correlation, there is but a single case where the power loss of the PDW with respect to the EDW is more than 1% (a,iv,100). However, in this particular case, the DW test has even a greater power loss. Moreover, the PDW is regularly more powerful than the feasible DW test or even the infeasible EDW test. The gains tend to increase with sample size and, consistent with our earlier findings, are the largest for error distributions with heavy tails. Thus, the PDW clearly dominates the DW test against these alternatives.

Moderate serial correlation: When there is moderate serial correlation in the data set, the results are equally decisive for the 5% and 10% level tests: losses are negligible and gains are occasionally large for heavy-tailed distributions. However, there is some loss of power at the 1% level test (the largest loss observed was 3.1%), when the DW test works well.

High serial correlation: Finally, when there is high serial correlation, all the tests quickly achieve maximal power; however, for $T = 12$, the PDW incurs serious power losses as compared to the DW at the 1% level tests.

Last, we consider the BDW test as an alternative to the PDW test. The results are directly related to those found for the sizes of the two tests. That is, for $T \geq 20$ the PDW has equal or better power than the BDW test, while at $T = 12$, the BDW occasionally has a small power advantage, (no greater than 3%). The superior power of the PDW test is particular striking when testing large samples of heavy-tailed distributions against limited serial correlation. For instance, for design (a) with sample size $T = 100$, the 10% PDW test has respectively 4%, 9%, and 15% higher power than the BDW for the following numbered distributions: (v) stable, (vii) the heavy-tailed Tukey Lambda, and (iv) the Cauchy distribution.

3.3. Resampling size

One might question the results of the size and power analyses presented above on two grounds. First, the observed power loss at the 1% level for the PDW test (versus the DW test) against moderate and high serial correlation could be due to the small number of permutations used by the PDW. Although in applied work, computer power will undoubtedly be sufficient to increase the number of resamples substantially, the large numbers of simulations and experiments prevented us from doing so in this study. We therefore concentrated our efforts on those cases where the PDW incurred a severe power loss. In particular, we repeated the power simulations for design (a) with the normally distributed errors (i) using sample sizes of 12, 20, and 50, all under moderate and high serial correlation; and by increasing the number of permutations from 199 to 999. Table 3 shows that the power losses decreased, such that the maximum power loss was less than 1.5%.

Second, the sometimes poor performance of the BDW (versus the PDW) could be due to the fact that the BDW is more sensitive to the small number of resamples. To consider this possibility, we focused on cases where the BDW performed poorly, as compared to the PDW. We chose to resample using the Cauchy distribution for the errors, the design matrix (a) and a large number of observations (50 and 100). We compared the 10% level tests of the PDW and the BDW against cases of limited serial correlation (0.1) with resampling sizes of 199 and 999. Table 4 presents the results and demonstrates that choosing a low resampling size did not cause the BDW to perform poorly.

4. Concluding remarks

We have shown that, for sample sizes larger than 20, the permutation test based on the Durbin-Watson (DW) statistic controls size better than the DW test for error distributions with heavy tails. When the DW test is conservative, this can result in large power gains at a very low cost: power losses are less than 1.5% when the Durbin-Watson test is correctly sized. A seemingly similar bootstrap procedure actually performs significantly worse than the permutation Durbin-Watson (PDW) test, and particularly when it matters most: in the case of heavy-tailed distributions.

These results suggest two promising areas for investigation, both of which are topics of the author's ongoing research. The first area concerns whether permutation tests may be applied to other testing problems and procedures. In particular, we believe it likely that permutation tests can be employed in conjunction with tests for higher-order serial correlation and heteroscedasticity.

Second, as the theory presented in this paper is new, it is admittedly underdeveloped, in particular when compared to the advances in bootstrap theory. Consequently, the following questions deserve investigation: (i) are permutation tests that use regression residuals asymptotically equal in power to their parametric counterparts?; (ii) can one obtain a result similar to that of the bootstrap test, in terms of the importance of pivotal statistics?; and (iii) is it possible to relax the moment restrictions used in this paper?

Appendix

A.1. Proof Theorem 1

We will show the slightly stronger result that under the conditions stated in the theorem

$$\lim_{T\to\infty} E\left(\left|p_T(r) - p_T(e)\right|\right) = 0 \tag{A-1}$$

which implies convergence in probability.

$$\begin{aligned}
& \lim_{T\to\infty} E\left|p_T(r) - p_T(e)\right| \\
= & \lim_{T\to\infty} E\left|\frac{1}{1+m}\left[\sum_{j=1}^{m} I\left\{\theta_T(Z_j r) \leq \theta_T(r)\right\} - \sum_{j=1}^{m} I\left\{\theta_T(Z_j e) \leq \theta_T(e)\right\}\right]\right| \\
= & \lim_{T\to\infty} E\left|\frac{1}{1+m}\sum_{j=1}^{m}\left[I\left\{\theta_T(Z_j r) \leq \theta_T(r) \leq \theta_T(Z_j e)\right\} - I\left\{\theta_T(Z_j e) \leq \theta_T(e) \leq \theta_T(Z_j r)\right\}\right]\right| \\
\leq & \lim_{T\to\infty} E\left(\frac{1}{1+m}\sum_{j=1}^{m} I\left\{\theta_T(Z_j r) \leq \theta_T(r) \leq \theta_T(Z_j e)\right\} + I\left\{\theta_T(Z_j e) \leq \theta_T(e) \leq \theta_T(Z_j r)\right\}\right) \\
= & \lim_{T\to\infty} \frac{1}{1+m}\sum_{j=1}^{m} P\left\{\theta_T(Z_j r) \leq \theta_T(r) \leq \theta_T(Z_j e)\right\} + P\left\{\theta_T(Z_j e) \leq \theta_T(e) \leq \theta_T(Z_j r)\right\} \\
= & \lim_{T\to\infty} \frac{m}{1+m}\left[P\left\{\theta_T(Zr) \leq \theta_T(r) \leq \theta_T(Ze)\right\} + P\left\{\theta_T(Ze) \leq \theta_T(e) \leq \theta_T(Zr)\right\}\right] \\
= & \frac{m}{m+1}\left[\lim_{T\to\infty} P\left\{\theta_T(Zr) \leq \theta_T(r) \leq \theta_T(Ze)\right\} + \lim_{T\to\infty} P\left\{\theta_T(Ze) \leq \theta_T(e) \leq \theta_T(Zr)\right\}\right] \\
= & \; 0
\end{aligned} \tag{A-2}$$

where the last equality follows by the assumptions made in the theorem.

A.2. Proof Theorem 2

We will prove that under the conditions of the theorem

$$\lim_{T\to\infty} P\left\{\theta_T\left(Ze\right)\leq\theta_T\left(e\right)\leq\theta_T\left(Zr\right)\right\} = 0 \tag{A-3}$$

The proof that

$$\lim_{T\to\infty} P\left\{\theta_T\left(Zr\right)\leq\theta_T\left(e\right)\leq\theta_T\left(Ze\right)\right\} = 0 \tag{A-4}$$

is similar.

$$\begin{aligned}
& \lim_{T\to\infty} P\left\{\theta_T\left(Ze\right)\leq\theta_T\left(e\right)\leq\theta_T\left(Zr\right)\right\} \\
= & \lim_{T\to\infty} P\left\{\theta_T\left(e\right)\geq\theta_T\left(Ze\right) \text{ and } \theta_T\left(e\right)\leq\theta_T\left(Zr\right)\right\} \\
= & \lim_{T\to\infty} P\left\{\begin{array}{c} g\left(T\right)\left(\theta_T\left(e\right)-\theta_T\left(Ze\right)\right)\geq 0 \text{ and} \\ g\left(T\right)\left(\theta_T\left(e\right)-\theta_T\left(Ze\right)\right)+g\left(T\right)\left(\theta_T\left(Ze\right)-\theta_T\left(Zr\right)\right)\leq 0 \end{array}\right\} \\
= & \lim_{T\to\infty} P\left\{Y\geq 0 \text{ and } Y\leq -g\left(T\right)\left(\theta_T\left(Ze\right)-\theta_T\left(Zr\right)\right)\right\}
\end{aligned} \tag{A-5}$$

where $Y = g\left(T\right)\left(\theta_T\left(e\right)-\theta_T\left(Ze\right)\right)$, which we have assumed has asymptotically a negligible probability to fall in a small interval around zero. Then

$$\begin{aligned}
& \lim_{T\to\infty} P\left\{Y\geq 0 \text{ and } Y\leq -g\left(T\right)\left(\theta_T\left(Ze\right)-\theta_T\left(Zr\right)\right)\right\} \\
\leq & \lim_{T\to\infty} P\left\{\left|Y\right|\leq\left|g\left(T\right)\left(\theta_T\left(Ze\right)-\theta_T\left(Zr\right)\right)\right|\right\} \\
= & 1-\lim_{T\to\infty} P\left\{\left|g\left(T\right)\left(\theta_T\left(Ze\right)-\theta_T\left(Zr\right)\right)\right|<\left|Y\right|\right\} \\
\leq & 1-\lim_{T\to\infty} P\left\{\left|g\left(T\right)\left(\theta_T\left(Ze\right)-\theta_T\left(Zr\right)\right)\right|<\varepsilon\right\}+\lim_{T\to\infty} P\left\{\left|Y\right|<\varepsilon\right\} \text{ for any } \varepsilon \\
= & \lim_{T\to\infty} P\left\{\left|Y\right|<\varepsilon\right\} \text{ for any } \varepsilon
\end{aligned} \tag{A-6}$$

To go from the third to the fourth line, we have used the inequality

$$P\left(A<B\right)\geq P\left(A<\varepsilon\right)-P\left(B<\varepsilon\right) \text{ for any } \varepsilon \tag{A-7}$$

Letting ε approach zero proves the theorem.

A.3. Lemmas to be used in proof of Theorem 3

In all these lemmas, we assume that e is an $T \times 1$ vector of iid random variables with $E(e_t) = 0$, and $E(e_t^2) = \sigma^2 < \infty$.

Lemma 1 *Let $A_T = (a_{ts})_{1\leq t\leq T, 1\leq s\leq T}$ be a sequence of positive semi-definite (p.s.d.) $T \times T$ matrices with the trace of A_T satisfying $tr(A_T) \leq a < \infty$. Then $e'A_Te = O_p(1)$.*

Proof. We want to show that for any $\varepsilon > 0$, there exists a constant C_ε such that $P\{|e'A_Te| \geq C_\varepsilon\} < \varepsilon$ for all T. Pick an ε and let C_ε be $2 \cdot a\sigma^2/\varepsilon$.

$$P\left\{\left|e'A_Te\right| \geq C_\varepsilon\right\} \leq \frac{E(|e'A_Te|)}{C_\varepsilon} = \frac{E(e'A_Te)}{2a\sigma^2/\varepsilon} = \frac{tr(A)\,\sigma^2\varepsilon}{2a\sigma^2} \leq \frac{\varepsilon}{2} < \varepsilon \qquad \text{(A-8)}$$

where we have used the generalized Chebyshev inequality, the fact that $|e'A_Te| = e'A_Te$ since A_T is p.s.d., and the following inequality on $E(e'A_Te)$

$$E\left(e'A_Te\right) = E\left(\sum_{t=1}^{T}\sum_{s=1}^{T} a_{ts}e_te_s\right) = E\left(\sum_{t=1}^{T} a_{tt}e_t^2\right) = tr(A)\,\sigma^2 \leq a\sigma^2 \qquad \text{(A-9)}$$

■

Lemma 2 *Let A be a p.s.d. $T \times T$ matrix and let Z be a $T \times T$ permutation matrix. Then*

1. $Z'AZ$ is p.s.d.
2. $tr(Z'AZ) = tr(A)$
3. $\lambda_{\max}(Z'AZ) = \lambda_{\max}(A)$ where $\lambda_{\max}(A)$ denotes the largest eigenvalue of A

Proof. Note that $Z' = Z^{-1}$, so that the eigenvalues of $Z'AZ$ are equal to the eigenvalues of A. All three results follow immediately. ■

Lemma 3 *Let X be a symmetric real $T \times T$ matrix and let Y be a symmetric p.s.d. $T \times T$ matrix. Then $tr(XY) \leq \lambda_{\max}(X) \cdot tr(Y)$.*

Proof. Directly from Lemma 1 in Wang, Kuo and Hsu (1986). ■

Lemma 4 *Three well-known properties of $P = X(X'X)^{-1}X'$ are:*

1. P is p.s.d.
2. $tr(P) = k$ where k is the number of columns of X
3. all eigenvalues of P are either 0 or 1, so that $\lambda_{\max}(P) = 1$

Lemma 5 *Some properties of W are:*

1. W is p.s.d.
2. $\lambda_{\max} \leq 4$

Proof. The first one is trivial to see from $e'We = \sum_{t=2}^{T}(e_t - e_{t-1})^2$. Using a well-known inequality on matrix norms, we get $|\lambda_{\max}| \leq \|W\|_1 = \max_s \sum_{t=1}^{T}|w_{ts}| = 4$. ■

Lemma 6 *The following random variables are all $O_p(1)$*

1. $e'Pe$
2. $e'PZ'PZe = e'Z'PZPe$
3. $e'PZ'PZPe$
4. $e'WPe = e'PWe$
5. $e'PWPe$
6. $e'PZ'WZe = e'Z'WZPe$
7. $e'PZ'PWZe = e'Z'WPZPe$
8. $e'PZ'WPZe = e'Z'PWZPe$
9. $e'PZ'PWPZe = e'Z'PWPZPe$
10. $e'PZ'WZPe$
11. $e'PZ'PWZPe = e'PZ'WPZPe$
12. $e'PZ'PWPZPe$

Proof. All follow straightforwardly from Lemmas 1, 2, 3, 4, and 5. To illustrate this, consider 11. We verify both conditions of Lemma 1. First, we show that $PZ'WPZP$ is p.s.d. Since W and P are p.s.d., so is WP and by Lemma 2 also $Z'WPZ$. The conclusion that $PZ'WPZP$ is p.s.d. follows directly. Second, we show that the $tr(PZ'WPZP)$ is bounded.

$$\begin{aligned} tr\left(PZ'WPZP\right) &= tr\left(WPZPZ'\right) \leq \lambda_{\max}(W)\, tr\left(PZPZ'\right) \\ &\leq 4 \cdot \lambda_{\max}(P) \cdot tr\left(ZPZ'\right) = 4tr(P) = 4k \qquad \text{(A-10)} \end{aligned}$$

■

Lemma 7 *The asymptotic behavior of three quadratic forms:*

1. $T^{-1}e'e \xrightarrow{p} \sigma^2$.
2. $T^{-1}e'We \xrightarrow{p} \sigma^2$.
3. $e'(W - 2I)e = O_p\left(T^{1/2}\right)$

Proof. The first follows directly from Komolgorov LLN. Note

$$\begin{aligned} e'We &= 2\sum_{t=1}^{T} e_t^2 - 2\sum_{t=2}^{T} e_t e_{t-1} - e_1^2 - e_T^2 \\ &= 2\sum_{t=1}^{T} e_t^2 - 2\sum_{t=1,3,5,\ldots}^{T-1} e_t e_{t+1} - 2\sum_{t=2,4,6,\ldots}^{T-1} e_t e_{t+1} + O_p(1) \end{aligned} \tag{A-11}$$

and

$$e'(W-2I)e = -2\sum_{t=1,3,5,\ldots}^{T-1} e_t e_{t+1} - 2\sum_{t=2,4,6,\ldots}^{T-1} e_t e_{t+1} + O_p(1) \tag{A-12}$$

Both $\sum_{t=1,3,5,\ldots}^{T-1} e_t e_{t+1}$ and $\sum_{t=2,4,6,\ldots}^{T-1} e_t e_{t+1}$ are sums of iid random variables with expectation 0 and variance σ^4. Results 2 and 3 follow then directly from Komolgorov LLN and Lindeberg-Levy CLT. ■

Lemma 8 *Using the previous two lemmas the following results are immediate:*

1. $T^{-1}e'Me = T^{-1}e'e + T^{-1}e'Pe \xrightarrow{p} \sigma^2$.
2. $T^{-1}e'Z'MZe \xrightarrow{p} \sigma^2$. Follows directly from the exchangeability of the e and the previous result.
3. $e'Z'MZPe = e'PZ'MZe = e'Z'ZPe + e'Z'PZPe = e'Pe + e'Z'PZPe = O_p(1) + O_p(1) = O_p(1)$.
4. $e'PZ'MZPe = e'Pe + e'PZ'PZPe = O_p(1) + O_p(1) = O_p(1)$.
5. $T^{-1}e'MZ'MZMe = T^{-1}e'Z'MZe - 2T^{-1}e'Z'MZPe + T^{-1}e'PZ'MZPe = T^{-1}e'Z'MZe + O_p\left(T^{-1}\right) \xrightarrow{p} \sigma^2$.
6. $T^{-1}e'MWMe = T^{-1}e'We - 2T^{-1}ePWe + T^{-1}e'PWPe \xrightarrow{p} 2\sigma^2$.
7. $T^{-1}e'Z'MWMZe \xrightarrow{p} 2\sigma^2$. By previous lemma and exchangeability.
8. $e'PZ'MWMZe = e'PZ'WZe - e'PZ'PWZe - e'PZ'WPZe + e'PZ'PWPZe = O_p(1)$
9. $e'PZ'MWMZPe = e'PZ'WZPe - 2e'PZ'PWZPe + e'PZ'PWPZPe = O_p(1)$
10. $T^{-1}e'MZ'MWMZMe \xrightarrow{p} 2\sigma^2$ by previous 3 results.
11. $e'(W-2I)Pe = e'WPe - 2e'Pe = O_p(1)$
12. $e'P(W-2I)Pe = e'PWPe - 2e'Pe = O_p(1)$.
13. $e'M(W-2I)Me = e'(W-2I)e - 2e'P(W-2I)e + e'P(W-2I)Pe = O_p\left(T^{1/2}\right)$
14. $e'Z'M(W-2I)MZe = O_p\left(T^{1/2}\right)$ by previous result and exchangeability.
15. $e'Z(W-2I)PZe = O_p(1)$ from 11 and exchangeability.
16. $e'Z'P(W-2I)PZe = O_p(1)$ from 12 and exchangeability.

A.4. Proof of Theorem 3

We start with the first condition of Theorem 2.

$$T^{1/2}\left(\theta_T(Zr) - \theta_T(Ze)\right)$$

$$
\begin{aligned}
&= T^{1/2}\left(\frac{e'MZ'MWMZMe}{e'MZ'MZMe}-\frac{e'Z'MWMZe}{e'Z'MZe}\right)\\
&= T^{1/2}\left(\frac{e'MZ'MWMZMe}{e'MZ'MZMe}-\frac{e'MZ'MWMZMe}{e'Z'MZe}\right)\\
&\quad +T^{1/2}\left(\frac{e'MZ'MWMZMe}{e'Z'MZe}-\frac{e'Z'MWMZe}{e'Z'MZe}\right)\\
&= \frac{\left(T^{-1}e'MZ'MWMZMe\right)T^{-1/2}\left\{e'Z'MZe-e'MZ'MZMe\right\}}{\left(T^{-1}e'MZ'MZMe\right)\left(T^{-1}e'Z'MZe\right)}\\
&\quad +\frac{T^{-1/2}\left(e'MZ'MWMZMe-e'Z'MWMZe\right)}{T^{-1}e'Z'MZe} \qquad \text{(A-13)}
\end{aligned}
$$

These pieces can be directly evaluated from Lemma 8. The denominators of the first and second fractions converge respectively to σ^4 and σ^2. The first piece of the numerator of the first fraction $T^{-1}e'MZ'MWMZMe = O_p(1)$. The second piece of numerator of the first fraction is

$$
\begin{aligned}
&T^{-1/2}\left\{e'Z'MZe-e'MZ'MZMe\right\}\\
&= 2T^{-1/2}e'PZ'MZe-T^{-1/2}e'PZ'MZPe\\
&= O_p\left(T^{-1/2}\right) \qquad \text{(A-14)}
\end{aligned}
$$

The numerator of the second fraction

$$
\begin{aligned}
&T^{-1/2}\left(e'MZ'MWMZMe-e'Z'MWMZe\right)\\
&= 2T^{-1/2}e'PZ'MWMZe-T^{-1/2}e'PZ'MWMZPe\\
&= O_p\left(T^{-1/2}\right) \qquad \text{(A-15)}
\end{aligned}
$$

Concluding

$$
\begin{aligned}
&T^{1/2}\left(\theta_T(Zr)-\theta_T(Ze)\right)\\
&= \frac{\left(T^{-1}e'MZ'MWMZMe\right)T^{-1/2}\left\{e'Z'MZe-e'MZ'MZMe\right\}}{\left(T^{-1}e'MZ'MZMe\right)\left(T^{-1}e'Z'MZe\right)}\\
&\quad +\frac{T^{-1/2}\left(e'MZ'MWMZMe-e'Z'MWMZe\right)}{T^{-1}e'Z'MZe}\\
&= \frac{O_p(1)\,O_p\left(T^{-1/2}\right)}{\sigma^4}+\frac{O_p\left(T^{-1/2}\right)}{\sigma^2}=O_p\left(T^{-1/2}\right) \qquad \text{(A-16)}
\end{aligned}
$$

Now the second condition of Theorem 2.

$$
\begin{aligned}
& T^{1/2}\left(\theta_T(e) - \theta_T(Ze)\right) \\
= \; & T^{1/2}\left(\frac{e'MWMe}{e'Me} - \frac{e'Z'MWMZe}{e'Z'MZe}\right) \\
= \; & T^{1/2}\left(\frac{e'MWMe}{e'Me} - \frac{2e'Me}{e'Me} - \frac{e'Z'MWMZe}{e'Z'MZe} + \frac{2e'Z'MZe}{e'Z'MZe}\right) \\
= \; & \frac{T^{-1/2}e'M(W-2I)Me}{T^{-1}e'Me} - \frac{T^{-1/2}e'Z'M(W-2I)MZe}{T^{-1}e'Z'MZe} \\
= \; & \frac{T^{-1/2}e'M(W-2I)Me}{T^{-1}e'Me} - \frac{T^{-1/2}e'Z'M(W-2I)MZe}{T^{-1}e'Me} \\
& + \frac{T^{-1/2}e'Z'M(W-2I)MZe}{T^{-1}e'Me} - \frac{T^{-1/2}e'Z'M(W-2I)MZe}{T^{-1}e'Z'MZe} \\
= \; & \frac{T^{-1/2}\left\{e'M(W-2I)Me - e'Z'M(W-2I)MZe\right\}}{T^{-1}e'Me} \\
& + \frac{\left\{T^{-1/2}e'Z'M(W-2I)MZe\right\}\left\{T^{-1}e'Z'MZe - T^{-1}e'Me\right\}}{\left(T^{-1}e'Me\right)\left(T^{-1}e'Z'MZe\right)}
\end{aligned}
\tag{A-17}
$$

Lemma 8 provides the components to evaluate this result. It shows that the second fraction converges to zero. The denominator of the first fraction converges to σ^2. The numerator of the first fraction is at most of order $O_p(1)$. We will show that it converges to a nondegenerate normal random variable so that the second condition of Theorem 2 is satisfied.

$$
\begin{aligned}
& T^{-1/2}\left\{e'M(W-2I)Me - e'Z'M(W-2I)MZe\right\} \\
= \; & T^{-1/2}e'(W-2I)e - T^{-1/2}e'Z'(W-2I)Ze + O\left(T^{-1/2}\right) \\
= \; & T^{-1/2}e'Be - T^{-1/2}e'Z'BZe + O_p\left(T^{-1/2}\right)
\end{aligned}
\tag{A-18}
$$

where

$$
B = \begin{bmatrix}
0 & -1 & 0 & . & . & . & . & 0 \\
-1 & 0 & -1 & & & & & 0 \\
0 & -1 & 0 & & & & & 0 \\
. & & & . & & & & . \\
. & & & & . & & & . \\
. & & & & & . & & . \\
. & & & & & & 0 & -1 \\
0 & 0 & 0 & . & . & . & -1 & 0
\end{bmatrix}
\tag{A-19}
$$

Consider

$$e'Be - T^{-1/2}e'Z'BZe$$

$$
\begin{aligned}
&= e'\left(B - Z'BZ\right)e \\
&= e'\tilde{B}e && \text{(A-20)}
\end{aligned}
$$

We use the Central Limit Theorem for quadratic forms of iid random variables of Rotar' (1975) (see also Guttorp and Lockhart, 1988; de Jong, 1987, Theorem 5.2). We note that $\tilde{B}$ is symmetric with $\tilde{b}_{tt} = 0$ for all t. Let μ_t be the eigenvalues of $\tilde{B}$ and $\sigma(T)^2 = Var\left(e'\tilde{B}e\right)$. If $\sigma(T)^{-2}\max_{1\le t\le T}\mu_t^2 \to 0$ as $T \to \infty$, then $\sigma(T)^{-1}e'\tilde{B}e \xrightarrow{d} N(0,1)$. We first note that $\max_{1\le t\le T}|\mu_t| \le \left\|\tilde{B}\right\|_1 = \max_s \sum_{t=1}^{T}\left|\tilde{b}_{ts}\right| = 4$, so that $\max_{1\le t\le T}\mu_t^2 \le 16$. We will now show that $\sigma(T)^2 \to \infty$.

Let Z be a permutation matrix; then $z(t)$ is the corresponding one-to-one permutation function that maps $\{1,2,...,T\}$ onto $\{1,2,...,T\}$ and $z'(t)$ is its inverse. Note that

$$
\begin{aligned}
e'\tilde{B}e &= -2\sum_{t=2}^{T} e_t e_{t-1} + 2\sum_{t=2}^{T} e_{z^0(t)} e_{z^0(t-1)} \\
&= -2\sum_{t=2}^{T} e_t e_{t-1} + 2\sum_{t=1,\ldots T;z(t)\neq 1}^{T} e_t e_{z^0(z(t)-1)} \\
&= -2\sum_{t=2}^{T} e_t e_{t-1} + 2\sum_{t=1,\ldots T;z(t)\neq 1}^{T} e_t e_{z^0(z(t)-1)} \\
&= 2\sum_{t=2,\ldots T;z(t)\neq 1} e_t\left(e_{z^0(z(t)-1)} - e_{t-1}\right) + O_p(1) && \text{(A-21)}
\end{aligned}
$$

The variance of the first term is

$$
\begin{aligned}
& E\left[\left(2\sum_{t=2,\ldots T;z(t)\neq 1} e_t\left(e_{z^0(z(t)-1)} - e_{t-1}\right)\right)^2\right] \\
\ge\ & 4\sum_{t=2,\ldots T;z(t)\neq 1} E\left[e_t\left(e_{z^0(z(t)-1)} - e_{t-1}\right)\right]^2 \\
=\ & 4\sum_{t=2,\ldots T;z(t)\neq 1} E\left(e_t^2\right)E\left(e_{z^0(z(t)-1)} - e_{t-1}\right)^2 \\
\ge\ & 4(T - v - 2)\,2\sigma^4 = 8\sigma^4 T\left(1 - \frac{v}{T} - \frac{2}{T}\right) && \text{(A-22)}
\end{aligned}
$$

where $v = \sum_{t=2,\ldots,T;z(t)\neq 1} I\{z'(z(t)-1) = t-1\}$; that is, v is the number of cases where two observations that are neighbors remain neighbors with the same ordering after the permutation has been applied. Since $\frac{v}{T} = o_p(1)$, the variance of $e'\tilde{B}e$ increases at least at a rate of T asymptotically.

Therefore, $\sigma(T)^{-2}\max_{1\le t\le T}\mu_t^2 \to 0$ as $T\to\infty$ and the central limit theorem holds. Furthermore, it must hold that $T^{-1/2}e'\tilde{B}e$ is nondegenerate asymptotically. The second condition of Theorem 2 follows directly.

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Table 1
Exact levels of DW, PDW, and BDW tests under various error distributions, design matrices, and sample sizes.

Design Matrix	Error Distribution	Nomimal Level	12 DW	12 PDW	12 BDW	20 DW	20 PDW	20 BDW	50 DW	50 PDW	50 BDW	100 DW	100 PDW	100 BDW
(a) constant	(i)	0.10	0.100	0.099	***0.108***	0.099	0.099	0.101	0.098	0.098	0.098	0.095	***0.094***	0.095
only	normal	0.05	0.052	0.053	0.054	0.049	0.050	0.052	0.049	0.049	0.049	0.049	0.048	0.048
		0.01	0.010	0.011	0.010	0.010	0.009	0.010	0.010	0.011	0.010	0.011	0.011	0.010
	(ii)	0.10	***0.107***	0.105	***0.113***	0.102	0.100	0.104	0.103	0.100	0.102	0.100	0.099	0.100
	uniform	0.05	***0.056***	0.052	***0.056***	0.052	0.049	0.050	0.052	0.052	0.050	0.053	0.053	0.050
		0.01	***0.012***	0.010	0.010	0.011	0.011	0.011	0.011	0.010	0.010	0.010	0.009	0.009
	(iii)	0.10	0.104	0.099	***0.108***	0.100	0.097	0.100	0.100	0.098	0.099	0.096	0.097	0.095
	lognormal	0.05	***0.055***	0.050	0.052	***0.057***	0.049	0.050	***0.060***	0.049	0.049	***0.061***	0.050	0.050
		0.01	***0.014***	0.010	0.012	***0.014***	0.010	***0.008***	***0.021***	0.011	0.010	***0.022***	0.012	0.012
	(iv)	0.10	***0.108***	0.096	***0.110***	0.097	0.101	0.099	***0.070***	0.103	***0.080***	***0.057***	0.104	***0.072***
	Cauchy	0.05	0.054	0.047	0.046	***0.064***	0.050	0.049	0.049	0.049	***0.044***	***0.040***	0.053	***0.040***
		0.01	***0.008***	0.010	***0.007***	0.012	0.009	***0.007***	***0.026***	0.010	***0.007***	***0.024***	***0.013***	***0.013***
	(v)	0.10	0.098	0.100	0.103	0.098	0.101	0.101	***0.089***	0.101	0.094	***0.083***	0.100	***0.091***
	stable	0.05	0.047	0.047	0.049	0.052	0.052	0.051	0.052	0.054	0.051	***0.045***	0.051	***0.045***
	a=1.6	0.01	***0.008***	0.009	***0.008***	***0.012***	0.011	0.009	***0.016***	0.011	0.009	***0.014***	0.011	0.010
	(vi)	0.10	0.101	0.101	***0.109***	0.096	0.103	0.100	***0.093***	0.103	0.097	***0.085***	0.096	***0.093***
	Tukey Lambda	0.05	0.048	0.052	0.051	0.052	0.052	0.049	0.049	0.052	0.048	0.046	0.048	0.047
	l1=l2=-1/3	0.01	0.008	0.008	0.008	0.010	0.010	0.008	***0.014***	0.011	0.009	0.011	0.009	0.010
	(vii)	0.10	0.104	0.104	***0.112***	0.101	0.104	0.103	***0.082***	0.104	***0.089***	***0.067***	0.099	***0.081***
	Tukey Lambda	0.05	0.051	0.049	0.050	***0.059***	***0.054***	0.050	0.051	***0.054***	0.049	***0.041***	0.048	***0.042***
	l1=l2=-2/3	0.01	***0.007***	0.008	***0.007***	0.010	0.009	***0.006***	***0.021***	0.011	0.008	***0.018***	0.012	0.011
(b) constant +	(i)	0.10	0.100	0.100	0.105	0.098	0.098	0.098	0.098	0.097	0.098	0.096	0.095	0.096
normal iid	normal	0.05	0.051	0.051	0.054	0.049	0.048	0.051	0.047	0.048	0.047	0.049	0.048	0.049
		0.01	0.011	0.010	0.011	0.010	0.011	0.009	0.009	0.011	0.010	0.011	0.011	0.011
	(ii)	0.10	***0.108***	***0.107***	***0.111***	0.100	0.100	0.102	0.102	0.100	0.100	0.101	0.100	0.101
	uniform	0.05	***0.056***	0.052	***0.056***	0.049	0.049	0.048	0.052	0.051	0.052	0.053	0.051	0.050
		0.01	***0.012***	0.011	0.011	0.011	0.010	0.010	0.010	0.010	0.010	0.010	0.009	0.010
	(iii)	0.10	0.095	0.096	0.104	0.101	0.098	0.099	0.098	0.098	0.098	0.096	0.097	***0.093***
	lognormal	0.05	***0.046***	0.048	0.053	***0.058***	0.051	0.050	***0.059***	0.049	0.049	***0.060***	0.050	0.051
		0.01	0.011	0.009	0.011	***0.014***	0.009	0.008	***0.020***	0.011	0.010	***0.022***	0.011	***0.013***
	(iv)	0.10	***0.087***	***0.091***	0.104	0.099	0.100	0.100	***0.071***	***0.094***	***0.080***	***0.057***	***0.092***	***0.071***
	Cauchy	0.05	***0.035***	***0.041***	0.046	***0.062***	0.051	0.051	0.049	0.050	***0.044***	***0.040***	0.049	***0.039***
		0.01	***0.007***	***0.007***	0.008	0.010	0.009	***0.006***	***0.026***	0.011	***0.006***	***0.024***	***0.015***	***0.014***
	(v)	0.10	***0.092***	0.094	0.100	0.097	0.097	0.098	***0.089***	0.100	0.094	***0.081***	0.098	***0.090***
	stable	0.05	***0.043***	0.046	0.048	0.053	0.050	0.052	0.052	0.054	0.051	***0.044***	0.049	0.046
	a=1.6	0.01	***0.007***	***0.007***	0.008	0.012	0.010	0.008	***0.016***	0.010	0.009	***0.014***	0.011	0.010
	(vi)	0.10	***0.093***	0.096	0.102	0.099	0.100	0.101	***0.092***	0.101	0.098	***0.085***	0.096	***0.093***
	Tukey Lambda	0.05	***0.044***	0.046	0.050	0.050	0.049	0.047	0.051	0.052	0.050	0.046	0.049	***0.045***
	l1=l2=-1/3	0.01	***0.006***	***0.007***	***0.008***	0.009	0.009	***0.008***	***0.013***	0.010	0.009	0.011	0.010	0.010
	(vii)	0.10	***0.090***	0.095	0.103	0.100	0.101	0.103	***0.082***	0.099	***0.087***	***0.067***	0.097	***0.081***
	Tukey Lambda	0.05	***0.038***	***0.044***	***0.045***	***0.056***	0.049	0.050	0.050	0.053	0.048	***0.041***	0.048	***0.042***
	l1=l2=-2/3	0.01	***0.005***	***0.006***	***0.005***	0.009	***0.008***	***0.006***	***0.022***	0.011	0.008	***0.019***	0.012	0.012

Table 1 (continued)

Design Matrix	Error Distribution	Nominal Level	12 DW	12 PDW	12 BDW	20 DW	20 PDW	20 BDW	50 DW	50 PDW	50 BDW	100 DW	100 PDW	100 BDW
(c) constant +	(i)	0.10	0.098	0.098	0.101	0.099	0.097	0.099	0.097	0.098	0.097	0.096	0.096	0.096
time	normal	0.05	0.052	0.049	0.052	0.051	0.049	0.050	0.047	0.049	0.046	0.048	0.048	0.049
		0.01	0.010	0.009	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.010
	(ii)	0.10	0.105	0.102	***0.109***	0.103	0.101	0.104	0.102	0.103	0.102	0.102	0.099	0.101
	uniform	0.05	***0.056***	0.052	***0.056***	0.053	0.050	0.051	0.051	0.052	0.050	0.052	0.053	0.052
		0.01	***0.013***	0.009	0.010	***0.012***	0.010	0.010	0.011	0.010	0.010	0.010	0.009	0.010
	(iii)	0.10	0.103	0.101	***0.107***	0.098	0.094	0.097	0.098	0.099	0.098	0.095	0.097	***0.094***
	lognormal	0.05	0.052	0.050	0.054	***0.055***	0.051	0.053	***0.058***	0.051	0.051	***0.061***	0.051	0.050
		0.01	0.011	0.009	***0.012***	***0.013***	0.010	0.011	***0.020***	0.012	0.010	***0.022***	0.012	***0.013***
	(iv)	0.10	***0.109***	***0.111***	***0.112***	***0.094***	0.101	0.098	***0.069***	0.099	***0.078***	***0.057***	0.095	***0.069***
	Cauchy	0.05	0.047	0.054	***0.055***	***0.059***	***0.061***	***0.058***	0.047	0.050	***0.045***	***0.040***	0.052	***0.038***
		0.01	***0.006***	***0.007***	***0.008***	0.010	0.008	0.009	***0.025***	***0.008***	0.010	***0.024***	***0.015***	***0.014***
	(v)	0.10	0.094	0.098	0.101	0.096	0.099	0.100	***0.089***	0.100	***0.093***	***0.082***	0.100	***0.090***
	stable	0.05	0.046	0.049	0.049	0.053	0.053	0.052	0.050	0.053	0.048	***0.043***	0.049	***0.042***
	a=1.6	0.01	***0.007***	***0.008***	0.009	0.012	0.011	0.011	***0.015***	0.010	0.009	***0.014***	0.012	0.011
	(vi)	0.10	0.097	0.102	0.103	0.097	0.101	0.100	***0.094***	0.100	0.098	***0.085***	0.094	***0.092***
	Tukey Lambda	0.05	0.048	0.050	0.049	0.050	0.053	0.048	0.049	0.052	0.049	***0.045***	0.049	0.047
	l1=l2=-1/3	0.01	0.008	***0.008***	0.009	0.008	0.011	0.009	***0.014***	0.011	0.010	0.010	0.009	0.010
	(vii)	0.10	0.102	0.105	***0.109***	0.098	0.104	0.099	***0.084***	0.101	***0.090***	***0.065***	0.096	***0.079***
	Tukey Lambda	0.05	0.049	***0.055***	0.052	***0.055***	***0.058***	***0.055***	0.050	0.054	0.048	***0.040***	0.048	***0.041***
	l1=l2=-2/3	0.01	***0.007***	***0.007***	***0.007***	***0.008***	0.010	***0.007***	***0.021***	0.012	0.010	***0.018***	***0.013***	0.011
(d) constant +	(i)	0.10	0.098	0.102	0.104	0.097	0.095	0.099	0.096	0.099	0.097	0.095	0.094	0.095
time +	normal	0.05	0.049	0.054	***0.055***	0.048	0.047	0.050	0.049	0.049	0.047	0.048	0.048	0.048
unit root		0.01	0.011	0.011	***0.013***	0.011	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.010
	(ii)	0.10	0.105	***0.109***	***0.114***	0.102	0.102	0.103	0.104	0.103	0.101	0.102	0.097	0.099
	uniform	0.05	0.052	***0.058***	***0.056***	0.052	0.050	0.052	0.053	0.050	0.052	0.054	0.054	0.053
		0.01	0.011	0.011	***0.015***	***0.012***	0.012	0.011	0.011	0.010	0.010	0.009	0.009	0.008
	(iii)	0.10	***0.112***	***0.119***	***0.117***	0.102	0.098	0.104	0.098	0.098	0.097	0.094	0.096	0.094
	lognormal	0.05	***0.064***	***0.068***	***0.061***	***0.058***	0.054	***0.056***	***0.059***	0.051	0.052	***0.059***	0.050	0.051
		0.01	***0.014***	***0.015***	***0.014***	***0.014***	0.012	0.011	***0.020***	0.011	0.011	***0.022***	***0.012***	***0.013***
	(iv)	0.10	***0.123***	***0.128***	***0.125***	0.095	0.097	0.097	***0.071***	***0.091***	***0.077***	***0.056***	***0.090***	***0.068***
	Cauchy	0.05	***0.072***	***0.083***	***0.065***	***0.063***	***0.063***	***0.062***	0.046	0.049	***0.044***	***0.040***	0.048	***0.039***
		0.01	0.010	***0.012***	0.011	0.011	0.011	0.010	***0.024***	***0.007***	0.011	***0.023***	***0.016***	***0.014***
	(v)	0.10	0.105	***0.112***	***0.110***	0.095	0.096	0.098	***0.089***	0.100	***0.093***	***0.082***	0.098	***0.089***
	stable	0.05	0.054	***0.060***	0.054	0.053	0.051	0.054	0.049	0.051	0.047	***0.043***	0.048	***0.043***
	a=1.6	0.01	0.009	0.011	0.011	***0.013***	***0.013***	0.011	***0.014***	0.009	0.009	***0.014***	0.011	0.010
	(vi)	0.10	***0.107***	***0.115***	***0.114***	0.099	0.102	0.103	***0.094***	0.100	0.096	***0.086***	0.095	***0.092***
	Tukey Lambda	0.05	***0.057***	***0.064***	***0.056***	0.054	***0.054***	***0.055***	0.050	0.053	0.049	***0.044***	0.048	***0.045***
	l1=l2=-1/3	0.01	0.010	***0.013***	***0.012***	0.010	***0.012***	0.010	***0.013***	0.011	0.010	0.011	0.009	0.009
	(vii)	0.10	***0.115***	***0.121***	***0.118***	0.102	0.105	0.105	***0.083***	0.099	***0.090***	***0.064***	0.095	***0.078***
	Tukey Lambda	0.05	***0.065***	***0.071***	***0.061***	***0.059***	***0.060***	***0.062***	0.050	0.054	0.048	***0.039***	0.046	***0.040***
	l1=l2=-2/3	0.01	0.009	***0.014***	0.012	0.010	0.011	0.010	***0.021***	0.011	0.011	***0.018***	***0.012***	0.011

Note: entries are exact levels for the indicated nominal test procedures. See text for further explanation.

Table 2.A

Power of EDW, DW, PDW, and BDW tests against AR(1) alternative with coefficient 0.1

			Sample size															
			12				20				50				100			
Design Matrix	Error Distribution	Nominal Level	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW
(a) constant only	(i) normal	0.10	0.157	0.001	0.002	-0.008	0.179	0.000	0.000	-0.002	0.266	-0.001	-0.002	-0.002	0.368	-0.003	0.000	0.000
		0.05	0.085	0.002	-0.001	-0.003	0.103	0.000	-0.001	-0.001	0.163	0.000	0.003	0.000	0.244	-0.003	0.002	0.001
		0.01	0.020	0.000	0.001	0.001	0.026	0.002	0.000	0.001	0.047	-0.001	0.001	0.005	0.086	0.002	0.009	0.006
	(ii) uniform	0.10	0.157	-0.002	0.002	***-0.011***	0.183	-0.002	0.000	-0.004	0.276	0.000	0.004	0.004	0.372	-0.001	0.003	0.001
		0.05	0.085	-0.005	-0.003	-0.006	0.102	-0.004	0.002	-0.002	0.170	0.001	0.005	0.004	0.242	0.001	0.005	0.003
		0.01	0.019	-0.004	0.001	0.000	0.024	-0.002	0.002	-0.001	0.047	-0.002	-0.001	0.000	0.087	-0.002	0.005	0.008
	(iii) lognormal	0.10	0.150	-0.002	-0.002	-0.008	0.158	-0.007	-0.004	-0.005	0.223	-0.002	-0.009	-0.001	0.323	0.006	***-0.012***	0.000
		0.05	0.081	-0.005	0.001	-0.004	0.087	***-0.015***	-0.003	-0.004	0.121	***-0.027***	-0.005	-0.006	0.174	***-0.031***	-0.007	-0.004
		0.01	0.018	-0.004	0.000	0.000	0.021	-0.009	0.001	0.004	0.028	***-0.027***	-0.004	-0.003	0.043	***-0.042***	-0.003	-0.003
	(iv) Cauchy	0.10	0.144	***-0.012***	***-0.014***	***-0.017***	0.153	0.003	***-0.027***	-0.002	0.254	***0.094***	***-0.172***	***0.035***	0.707	***0.500***	***0.034***	***0.183***
		0.05	0.086	-0.006	-0.007	-0.006	0.081	***-0.019***	***-0.018***	-0.008	0.101	-0.001	***-0.073***	0.005	0.180	***0.061***	***-0.203***	***0.023***
		0.01	0.020	0.006	-0.007	0.006	0.033	-0.002	0.001	***0.021***	0.019	***-0.029***	***-0.016***	-0.008	0.018	***-0.035***	***-0.029***	***-0.013***
	(v) stable a=1.6	0.10	0.151	0.002	0.000	-0.007	0.176	0.005	-0.001	0.001	0.266	***0.030***	***-0.037***	***0.012***	0.396	***0.061***	***-0.042***	0.003
		0.05	0.081	0.001	-0.003	-0.003	0.099	-0.002	-0.004	0.001	0.149	0.009	***-0.011***	0.008	0.233	***0.024***	***-0.043***	***0.019***
		0.01	0.019	0.003	0.000	0.003	0.027	-0.001	0.000	0.005	0.033	***-0.017***	-0.008	-0.004	0.049	***-0.026***	***-0.016***	***-0.012***
	(vi) Tukey Lambda l1=l2=-1/3	0.10	0.156	0.005	-0.005	-0.009	0.182	0.007	-0.003	0.000	0.274	***0.023***	-0.006	0.008	0.386	***0.030***	-0.005	0.009
		0.05	0.085	0.005	0.000	0.002	0.098	0.000	-0.004	0.001	0.157	0.006	0.000	0.009	0.235	***0.012***	0.000	***0.015***
		0.01	0.020	0.004	0.002	0.004	0.024	-0.001	-0.001	0.003	0.039	-0.010	-0.004	-0.001	0.057	***-0.016***	-0.008	-0.003
	(vii) Tukey Lambda l1=l2=-2/3	0.10	0.152	-0.002	-0.008	-0.008	0.168	0.006	***-0.013***	-0.001	0.261	***0.057***	***-0.078***	***0.023***	0.427	***0.142***	***-0.087***	-0.001
		0.05	0.086	-0.001	-0.005	-0.002	0.088	***-0.012***	***-0.014***	-0.006	0.126	0.000	***-0.028***	0.008	0.195	***0.030***	***-0.084***	***0.018***
		0.01	0.020	0.005	-0.001	0.006	0.026	-0.003	0.001	***0.010***	0.023	***-0.026***	***-0.016***	-0.007	0.024	***-0.035***	***-0.020***	***-0.015***
(b) constant + normal iid	(i) normal	0.10	0.153	0.002	0.001	-0.003	0.173	0.000	0.002	-0.001	0.263	0.000	0.000	-0.003	0.364	-0.003	-0.003	0.000
		0.05	0.083	0.002	0.001	0.000	0.098	0.001	0.002	0.001	0.158	-0.002	0.001	0.000	0.239	-0.003	-0.001	0.002
		0.01	0.019	0.000	0.000	-0.001	0.026	0.002	0.001	0.003	0.044	-0.001	0.000	0.004	0.085	0.002	0.006	0.006
	(ii) uniform	0.10	0.156	-0.005	-0.002	-0.007	0.179	-0.001	0.001	-0.002	0.270	0.001	0.003	0.002	0.371	0.000	0.007	0.003
		0.05	0.083	-0.005	-0.003	-0.006	0.098	-0.003	0.002	0.000	0.166	0.001	0.004	0.004	0.242	0.001	0.007	0.004
		0.01	0.018	-0.004	-0.001	-0.002	0.022	-0.002	-0.001	-0.001	0.044	-0.003	-0.001	-0.001	0.085	-0.003	0.004	0.005
	(iii) lognormal	0.10	0.150	0.009	0.005	-0.004	0.155	-0.009	-0.006	-0.009	0.222	-0.002	-0.008	0.000	0.321	0.004	-0.009	0.002
		0.05	0.081	0.006	0.003	-0.003	0.086	***-0.012***	-0.002	-0.004	0.119	***-0.024***	-0.004	-0.006	0.173	***-0.031***	-0.009	-0.006
		0.01	0.019	0.001	0.002	0.001	0.021	-0.008	-0.001	0.003	0.027	***-0.027***	-0.006	-0.003	0.043	***-0.041***	-0.005	-0.006
	(iv) Cauchy	0.10	0.153	***0.019***	***0.010***	-0.002	0.155	-0.001	-0.008	-0.006	0.249	***0.090***	***-0.121***	***0.051***	0.661	***0.450***	***0.017***	***0.180***
		0.05	0.088	***0.017***	0.008	0.004	0.083	***-0.019***	***-0.014***	***-0.011***	0.100	-0.002	***-0.040***	0.004	0.181	***0.062***	***-0.178***	***0.036***
		0.01	0.022	0.010	0.005	0.004	0.033	0.000	0.005	***0.021***	0.019	***-0.028***	***-0.015***	-0.007	0.018	***-0.034***	***-0.028***	***-0.013***
	(v) stable a=1.6	0.10	0.149	0.008	0.003	-0.003	0.173	0.006	0.002	0.002	0.259	***0.030***	***-0.025***	***0.014***	0.392	***0.056***	***-0.039***	0.004
		0.05	0.080	0.006	0.004	0.001	0.095	-0.003	-0.001	-0.003	0.145	0.005	-0.008	0.008	0.228	***0.023***	***-0.041***	***0.015***
		0.01	0.018	0.005	0.003	0.002	0.027	0.000	0.002	0.006	0.032	***-0.018***	***-0.011***	-0.005	0.048	***-0.024***	***-0.016***	***-0.011***
	(vi) Tukey Lambda l1=l2=-1/3	0.10	0.159	***0.013***	0.007	-0.002	0.175	0.003	-0.003	-0.004	0.268	***0.017***	-0.004	0.007	0.381	***0.025***	-0.005	***0.010***
		0.05	0.084	***0.011***	0.005	0.002	0.095	-0.002	-0.002	-0.002	0.151	0.004	-0.003	0.004	0.234	***0.012***	-0.001	***0.013***
		0.01	0.017	0.004	0.002	0.000	0.021	-0.001	0.001	0.002	0.039	-0.008	-0.003	0.002	0.058	***-0.016***	-0.008	-0.002
	(vii) Tukey Lambda l1=l2=-2/3	0.10	0.152	***0.015***	0.006	-0.004	0.170	0.003	-0.004	-0.003	0.256	***0.052***	***-0.056***	***0.026***	0.429	***0.141***	***-0.072***	***0.019***
		0.05	0.088	***0.017***	0.010	0.007	0.086	***-0.013***	***-0.011***	-0.009	0.125	0.001	***-0.015***	0.004	0.198	***0.032***	***-0.072***	***0.021***
		0.01	0.020	***0.010***	0.006	0.007	0.023	-0.002	0.002	0.008	0.022	***-0.027***	***-0.015***	-0.008	0.024	***-0.035***	***-0.020***	***-0.015***

Table 2.A (continued)

			Sample size															
			12				20				50				100			
Design Matrix	Error Distribution	Nominal Level	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW
(c) constant +	(i)	0.10	0.144	0.001	0.000	-0.007	0.170	0.000	0.001	-0.001	0.263	-0.001	-0.002	-0.001	0.365	-0.004	-0.003	-0.001
time	normal	0.05	0.081	0.002	0.002	0.000	0.101	0.001	0.005	0.005	0.158	0.000	0.001	0.003	0.246	-0.001	0.003	0.005
		0.01	0.018	0.000	0.002	0.000	0.028	0.002	0.003	0.003	0.045	0.000	0.002	0.003	0.083	0.001	0.008	0.005
	(ii)	0.10	0.151	-0.007	-0.001	-0.006	0.182	-0.003	0.003	-0.001	0.274	0.001	0.005	0.006	0.367	-0.001	0.002	0.003
	uniform	0.05	0.079	-0.006	0.001	-0.005	0.098	-0.005	-0.002	-0.002	0.167	0.001	0.006	0.004	0.241	0.000	0.005	0.004
		0.01	0.017	-0.004	0.002	0.000	0.024	-0.003	-0.001	0.001	0.046	-0.002	-0.001	0.001	0.087	0.001	0.008	0.006
	(iii)	0.10	0.144	-0.003	-0.004	-0.006	0.151	-0.006	-0.007	-0.005	0.221	0.000	-0.008	-0.001	0.324	0.006	-0.009	0.001
	lognormal	0.05	0.079	-0.003	0.001	-0.004	0.082	***-0.013***	-0.005	-0.008	0.120	***-0.024***	-0.005	-0.007	0.172	***-0.030***	-0.009	-0.006
		0.01	0.017	-0.002	0.002	-0.003	0.019	-0.008	-0.001	-0.001	0.027	***-0.027***	-0.006	-0.005	0.044	***-0.040***	-0.004	-0.005
	(iv)	0.10	0.145	-0.007	***-0.013***	***-0.016***	0.153	0.007	***-0.021***	0.002	0.255	***0.095***	***-0.125***	***0.043***	0.669	***0.462***	***0.014***	***0.196***
	Cauchy	0.05	0.087	0.002	-0.005	-0.003	0.079	***-0.014***	***-0.017***	***-0.012***	0.102	0.002	***-0.064***	0.004	0.183	***0.065***	***-0.196***	***0.043***
		0.01	0.020	0.007	0.006	0.004	0.032	0.001	0.008	***0.016***	0.019	***-0.027***	***-0.012***	***-0.010***	0.018	***-0.034***	***-0.026***	***-0.014***
	(v)	0.10	0.148	0.003	-0.001	-0.002	0.175	0.010	0.001	0.006	0.260	***0.026***	***-0.027***	0.010	0.392	***0.060***	***-0.038***	0.008
	stable	0.05	0.078	0.003	0.001	-0.003	0.098	0.001	0.000	0.002	0.145	0.007	***-0.011***	0.008	0.232	***0.023***	***-0.042***	***0.020***
	a=1.6	0.01	0.018	0.003	0.003	0.003	0.028	0.001	0.002	0.005	0.033	***-0.013***	-0.008	-0.005	0.048	***-0.024***	***-0.014***	-0.009
	(vi)	0.10	0.152	0.004	-0.001	-0.003	0.179	0.007	0.000	0.002	0.269	***0.022***	-0.010	0.008	0.381	***0.026***	-0.008	0.009
	Tukey Lambda	0.05	0.079	0.005	-0.001	0.000	0.097	0.003	-0.001	0.002	0.156	0.007	0.000	0.010	0.232	***0.013***	-0.004	***0.011***
	l1=l2=-1/3	0.01	0.018	0.002	0.003	0.000	0.023	0.001	-0.001	0.004	0.040	-0.007	-0.004	0.000	0.060	***-0.013***	-0.004	0.000
	(vii)	0.10	0.149	0.002	-0.004	-0.007	0.168	0.008	-0.010	0.002	0.258	***0.054***	***-0.063***	***0.024***	0.431	***0.146***	***-0.076***	***0.022***
	Tukey Lambda	0.05	0.084	0.004	-0.001	0.004	0.088	-0.007	***-0.013***	-0.006	0.125	0.002	***-0.031***	0.004	0.197	***0.035***	***-0.083***	***0.022***
	l1=l2=-2/3	0.01	0.017	0.004	0.004	0.002	0.026	0.000	0.001	0.008	0.024	***-0.025***	***-0.014***	-0.008	0.024	***-0.034***	***-0.020***	***-0.013***
(d) constant +	(i)	0.10	0.134	-0.002	-0.009	***-0.014***	0.166	0.001	0.003	-0.002	0.257	-0.001	-0.002	0.000	0.366	0.000	0.004	0.003
time +	normal	0.05	0.073	0.000	-0.006	-0.006	0.094	0.001	0.004	-0.001	0.157	0.000	0.002	0.000	0.246	0.003	0.008	0.010
unit root		0.01	0.016	0.000	-0.001	-0.003	0.023	0.000	0.000	0.001	0.045	0.000	0.003	0.003	0.081	0.000	0.005	0.004
	(ii)	0.10	0.145	-0.002	-0.006	-0.008	0.177	-0.003	0.001	-0.002	0.267	0.001	0.003	0.004	0.364	-0.001	0.000	0.000
	uniform	0.05	0.077	-0.001	-0.007	-0.007	0.095	-0.003	-0.002	-0.002	0.167	0.003	0.009	0.007	0.241	0.001	0.006	0.005
		0.01	0.016	-0.002	-0.001	-0.006	0.024	-0.003	-0.001	0.000	0.046	-0.003	0.002	0.000	0.089	0.001	0.010	0.009
	(iii)	0.10	0.132	***-0.018***	***-0.023***	***-0.026***	0.150	-0.007	-0.006	-0.006	0.220	-0.001	-0.009	-0.002	0.320	0.006	-0.009	0.002
	lognormal	0.05	0.074	***-0.014***	***-0.019***	***-0.012***	0.082	***-0.016***	-0.006	-0.009	0.121	***-0.022***	-0.004	-0.005	0.173	***-0.030***	-0.007	-0.005
		0.01	0.017	-0.006	-0.007	-0.005	0.020	-0.007	-0.003	-0.002	0.027	***-0.025***	-0.008	-0.008	0.043	***-0.039***	-0.005	-0.005
	(iv)	0.10	0.139	***-0.020***	***-0.028***	***-0.028***	0.147	0.001	***-0.015***	-0.004	0.255	***0.096***	***-0.082***	***0.051***	0.649	***0.439***	***0.013***	***0.201***
	Cauchy	0.05	0.074	***-0.028***	***-0.038***	***-0.019***	0.076	***-0.019***	***-0.021***	***-0.018***	0.101	0.003	***-0.040***	0.004	0.182	***0.067***	***-0.186***	***0.046***
		0.01	0.029	0.002	-0.002	0.000	0.030	-0.003	-0.001	0.008	0.018	***-0.027***	***-0.011***	-0.009	0.018	***-0.033***	***-0.027***	***-0.015***
	(v)	0.10	0.143	-0.004	***-0.012***	-0.007	0.172	0.009	0.003	0.004	0.258	***0.026***	***-0.019***	***0.013***	0.388	***0.056***	***-0.032***	0.006
	stable	0.05	0.073	-0.007	***-0.014***	***-0.010***	0.091	-0.001	-0.001	-0.002	0.145	0.008	-0.004	0.009	0.228	***0.021***	***-0.039***	***0.016***
	a=1.6	0.01	0.018	0.000	-0.003	-0.002	0.026	0.000	0.000	0.004	0.032	***-0.011***	-0.006	-0.004	0.047	***-0.023***	***-0.015***	-0.010
	(vi)	0.10	0.142	-0.005	***-0.014***	***-0.014***	0.177	0.005	-0.001	0.002	0.263	***0.018***	-0.005	0.007	0.380	***0.029***	-0.004	0.009
	Tukey Lambda	0.05	0.076	-0.005	***-0.013***	-0.008	0.094	-0.001	-0.004	-0.001	0.155	***0.010***	0.001	***0.010***	0.231	***0.016***	-0.003	***0.014***
	l1=l2=-1/3	0.01	0.018	0.001	-0.003	-0.002	0.025	0.001	0.000	0.002	0.040	-0.006	-0.002	0.002	0.059	***-0.012***	-0.007	0.000
	(vii)	0.10	0.135	***-0.016***	***-0.023***	***-0.021***	0.165	0.004	-0.006	-0.002	0.255	***0.053***	***-0.042***	***0.024***	0.433	***0.146***	***-0.063***	***0.032***
	Tukey Lambda	0.05	0.074	***-0.017***	***-0.026***	***-0.013***	0.087	***-0.012***	***-0.012***	-0.010	0.127	0.003	***-0.021***	0.003	0.198	***0.034***	***-0.079***	***0.026***
	l1=l2=-2/3	0.01	0.021	0.000	-0.005	-0.001	0.024	-0.004	-0.003	0.002	0.023	***-0.024***	***-0.013***	-0.009	0.024	***-0.032***	***-0.020***	***-0.014***

Note: entries in the columns labelled EDW are the exact powers of the EDW test for the levels indicated.
Entries in the other columns are the differences between the power of the EDW and one of the DW, PDW, or BDW tests.
See text for further explanation.

Table 2.B

Power of EDW, DW, PDW, and BDW tests against AR(1) alternative with coefficient 0.5

			Sample size: 12				20				50				100			
Design Matrix	Error Distribution	Nominal Level	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW
(a) constant	(i)	0.10	0.544	0.002	0.002	***-0.013***	0.733	0.000	0.003	0.003	0.979	0.000	0.001	0.001	1.000	0.000	0.000	0.000
only	normal	0.05	0.407	0.003	0.007	-0.002	0.618	0.001	0.004	0.004	0.958	0.000	0.003	0.001	1.000	0.000	0.000	0.000
		0.01	0.183	0.001	***0.014***	***0.017***	0.384	0.009	***0.031***	***0.037***	0.864	-0.001	***0.023***	***0.023***	0.995	0.000	0.003	0.003
	(ii)	0.10	0.529	-0.004	0.000	***-0.013***	0.745	-0.002	0.001	-0.001	0.977	0.000	0.000	0.001	1.000	0.000	0.000	0.000
	uniform	0.05	0.383	***-0.013***	-0.005	***-0.017***	0.631	-0.007	0.002	0.001	0.955	0.000	0.003	0.003	0.999	0.000	0.000	0.000
		0.01	0.159	***-0.021***	-0.005	-0.007	0.368	***-0.019***	***0.011***	0.010	0.858	-0.006	***0.020***	***0.027***	0.995	0.000	0.002	0.002
	(iii)	0.10	0.537	-0.004	***-0.011***	***-0.028***	0.777	***-0.014***	-0.006	***-0.013***	0.995	0.000	0.001	0.001	1.000	0.000	0.000	0.000
	lognormal	0.05	0.386	***-0.010***	-0.005	***-0.018***	0.597	***-0.046***	***-0.025***	***-0.029***	0.974	-0.009	-0.003	-0.004	1.000	0.000	0.000	0.000
		0.01	0.158	***-0.025***	-0.006	-0.004	0.290	***-0.065***	-0.004	-0.007	0.788	***-0.125***	***-0.019***	***-0.025***	0.996	-0.004	0.002	0.002
	(iv)	0.10	0.560	***-0.036***	***-0.074***	***-0.077***	0.869	0.004	0.006	0.003	0.993	0.005	0.003	0.004	0.998	0.000	0.000	0.000
	Cauchy	0.05	0.391	***-0.016***	***-0.046***	***-0.040***	0.676	***-0.083***	***-0.064***	***-0.043***	0.979	0.000	0.002	0.002	0.998	0.000	0.000	0.001
		0.01	0.221	***0.030***	***0.027***	***0.030***	0.350	***-0.011***	***0.020***	***0.030***	0.702	***-0.244***	***-0.130***	***-0.113***	0.966	***-0.028***	***-0.015***	-0.006
	(v)	0.10	0.565	0.003	-0.007	***-0.017***	0.790	0.010	0.009	0.007	0.985	0.002	0.003	0.002	1.000	0.000	0.000	0.000
	stable	0.05	0.416	0.004	-0.001	-0.010	0.664	-0.004	0.005	0.007	0.967	0.003	0.004	0.004	0.999	0.000	0.000	0.000
	a=1.6	0.01	0.196	***0.017***	***0.015***	***0.018***	0.378	-0.005	***0.022***	***0.029***	0.852	***-0.042***	***0.011***	***0.022***	0.993	-0.003	0.001	0.006
	(vi)	0.10	0.563	0.006	0.001	***-0.013***	0.794	0.008	0.006	0.005	0.983	0.001	0.002	0.002	0.999	0.000	0.000	0.000
	Tukey Lambda	0.05	0.420	***0.013***	0.006	-0.004	0.675	-0.001	0.005	0.010	0.966	0.002	0.004	0.003	0.999	0.000	0.000	0.000
	l1=l2=-1/3	0.01	0.202	***0.020***	***0.022***	***0.025***	0.386	-0.006	***0.014***	***0.024***	0.863	***-0.022***	***0.018***	***0.028***	0.994	-0.001	0.003	0.004
	(vii)	0.10	0.578	-0.003	***-0.021***	***-0.031***	0.839	0.007	***0.012***	0.005	0.987	0.004	0.003	0.004	0.999	0.001	0.000	0.000
	Tukey Lambda	0.05	0.403	-0.003	***-0.027***	***-0.026***	0.688	***-0.031***	***-0.020***	-0.007	0.972	0.001	0.004	0.002	0.998	0.001	0.000	0.001
	l1=l2=-2/3	0.01	0.206	***0.026***	***0.022***	***0.025***	0.360	***-0.020***	0.008	***0.023***	0.806	***-0.116***	***-0.027***	***-0.018***	0.982	***-0.011***	0.000	0.006
(b) constant +	(i)	0.10	0.516	0.002	0.005	-0.006	0.709	0.000	0.004	0.002	0.977	0.000	0.002	0.001	1.000	0.000	0.000	0.000
normal iid	normal	0.05	0.376	0.003	0.004	-0.004	0.589	0.000	0.004	0.005	0.951	0.000	0.002	0.001	1.000	0.000	0.000	0.000
		0.01	0.159	0.000	0.006	0.009	0.350	***0.011***	***0.031***	***0.037***	0.849	-0.002	***0.022***	***0.022***	0.995	0.000	0.003	0.004
	(ii)	0.10	0.495	-0.008	0.002	***-0.013***	0.724	0.000	0.006	0.001	0.974	0.000	0.001	0.001	0.999	0.000	0.000	0.000
	uniform	0.05	0.351	***-0.012***	-0.007	***-0.013***	0.605	-0.006	0.007	0.003	0.950	0.001	0.005	0.005	0.999	0.000	0.000	0.000
		0.01	0.141	***-0.019***	-0.006	-0.007	0.337	***-0.015***	0.003	***0.012***	0.841	-0.008	***0.017***	***0.023***	0.995	0.000	0.003	0.002
	(iii)	0.10	0.536	***0.017***	***0.014***	-0.003	0.741	***-0.013***	-0.005	***-0.012***	0.992	0.000	0.001	0.001	1.000	0.000	0.000	0.000
	lognormal	0.05	0.387	***0.016***	***0.018***	0.003	0.566	***-0.041***	***-0.018***	***-0.026***	0.969	-0.008	-0.002	-0.004	1.000	0.000	0.000	0.000
		0.01	0.167	0.004	***0.015***	0.009	0.270	***-0.056***	-0.004	***-0.011***	0.760	***-0.134***	***-0.032***	***-0.033***	0.995	-0.005	0.001	0.002
	(iv)	0.10	0.598	***0.041***	***0.015***	-0.002	0.838	-0.001	***0.013***	0.003	0.992	0.005	0.003	0.004	0.998	0.000	0.000	0.000
	Cauchy	0.05	0.420	***0.043***	***0.031***	-0.002	0.606	***-0.089***	***-0.056***	***-0.064***	0.978	0.000	0.003	0.002	0.997	0.000	0.000	0.001
		0.01	0.222	***0.052***	***0.054***	***0.041***	0.333	-0.004	***0.028***	***0.024***	0.610	***-0.328***	***-0.200***	***-0.177***	0.965	***-0.029***	***-0.016***	-0.004
	(v)	0.10	0.544	***0.015***	***0.012***	-0.002	0.765	0.005	0.009	0.007	0.983	0.003	0.003	0.003	1.000	0.000	0.000	0.000
	stable	0.05	0.402	***0.019***	***0.016***	0.005	0.622	-0.006	0.003	0.005	0.962	0.002	0.004	0.006	0.999	0.000	0.000	0.000
	a=1.6	0.01	0.183	***0.027***	***0.030***	***0.025***	0.346	-0.001	***0.026***	***0.029***	0.831	***-0.050***	0.006	***0.018***	0.992	-0.004	0.002	0.006
	(vi)	0.10	0.547	***0.018***	***0.017***	0.003	0.764	0.002	0.006	0.000	0.981	0.001	0.001	0.002	0.999	0.000	0.000	0.000
	Tukey Lambda	0.05	0.400	***0.023***	***0.020***	0.004	0.634	-0.003	0.005	0.008	0.962	0.001	0.003	0.004	0.999	0.000	0.000	0.000
	l1=l2=-1/3	0.01	0.181	***0.026***	***0.027***	***0.021***	0.354	-0.005	***0.018***	***0.026***	0.843	***-0.027***	***0.015***	***0.024***	0.993	-0.002	0.003	0.004
	(vii)	0.10	0.570	***0.027***	***0.012***	0.000	0.806	0.004	***0.012***	0.004	0.986	0.004	0.004	0.004	0.999	0.001	0.000	0.001
	Tukey Lambda	0.05	0.419	***0.041***	***0.035***	***0.014***	0.633	***-0.036***	***-0.016***	***-0.017***	0.970	0.000	0.005	0.002	0.998	0.000	0.000	0.001
	l1=l2=-2/3	0.01	0.208	***0.048***	***0.052***	***0.040***	0.333	***-0.014***	***0.018***	***0.015***	0.775	***-0.136***	***-0.043***	***-0.032***	0.981	***-0.012***	-0.001	0.006

Table 2.B (continued)

Design Matrix	Error Distribution	Nominal Level	12 EDW	12 EDW-DW	12 EDW-PDW	12 EDW-BDW	20 EDW	20 EDW-DW	20 EDW-PDW	20 EDW-BDW	50 EDW	50 EDW-DW	50 EDW-PDW	50 EDW-BDW	100 EDW	100 EDW-DW	100 EDW-PDW	100 EDW-BDW
(c) constant +	(i)	0.10	0.454	0.002	0.003	-0.003	0.670	0.000	0.004	0.005	0.974	0.000	0.001	0.001	1.000	0.000	0.000	0.000
time	normal	0.05	0.323	0.003	***0.011***	0.002	0.547	0.003	***0.011***	***0.013***	0.947	0.000	0.003	0.004	0.999	0.000	0.000	0.000
		0.01	0.126	0.002	***0.021***	***0.013***	0.307	0.008	***0.024***	***0.027***	0.838	0.000	***0.028***	***0.028***	0.993	0.000	0.004	0.005
	(ii)	0.10	0.444	-0.009	-0.003	***-0.012***	0.680	-0.004	0.001	0.000	0.971	0.000	0.001	0.003	1.000	0.000	0.000	0.000
	uniform	0.05	0.307	***-0.014***	-0.003	***-0.013***	0.558	***-0.010***	0.002	0.005	0.945	0.001	0.005	0.006	0.999	0.000	0.000	0.000
		0.01	0.114	***-0.017***	0.007	0.000	0.299	***-0.021***	-0.002	***0.012***	0.827	-0.007	***0.020***	***0.025***	0.994	0.000	0.003	0.003
	(iii)	0.10	0.451	-0.004	***-0.011***	***-0.019***	0.707	***-0.011***	-0.005	-0.009	0.990	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	lognormal	0.05	0.309	-0.006	-0.004	-0.009	0.526	***-0.036***	***-0.026***	***-0.019***	0.963	***-0.011***	-0.005	-0.005	1.000	0.000	0.000	0.000
		0.01	0.117	***-0.012***	0.008	-0.002	0.239	***-0.048***	***-0.014***	***-0.014***	0.748	***-0.133***	***-0.036***	***-0.039***	0.995	-0.005	0.001	0.002
	(iv)	0.10	0.486	***-0.015***	***-0.036***	***-0.046***	0.806	***0.013***	***0.013***	0.006	0.991	0.004	0.004	0.004	0.998	0.001	0.000	0.000
	Cauchy	0.05	0.332	0.005	***-0.022***	***-0.015***	0.578	***-0.068***	***-0.066***	***-0.056***	0.977	0.000	0.002	0.001	0.997	0.000	0.000	0.001
		0.01	0.170	***0.030***	***0.040***	***0.030***	0.308	0.009	***0.021***	***0.037***	0.633	***-0.298***	***-0.191***	***-0.176***	0.962	***-0.031***	***-0.020***	-0.010
	(v)	0.10	0.469	0.003	0.000	***-0.010***	0.735	***0.014***	***0.017***	***0.015***	0.981	0.004	0.004	0.003	1.000	0.000	0.000	0.000
	stable	0.05	0.333	0.008	0.008	0.002	0.586	0.001	0.007	***0.011***	0.959	0.003	0.004	0.007	0.999	0.000	0.000	0.000
	a=1.6	0.01	0.139	***0.011***	***0.024***	***0.018***	0.317	0.008	***0.023***	***0.032***	0.826	***-0.039***	0.008	***0.019***	0.992	-0.003	0.002	0.005
	(vi)	0.10	0.475	0.009	0.004	-0.003	0.740	0.009	***0.011***	***0.012***	0.978	0.002	0.002	0.002	0.999	0.000	0.000	0.000
	Tukey Lambda	0.05	0.335	0.010	0.006	0.004	0.601	0.006	0.010	***0.017***	0.956	0.002	0.003	0.004	0.999	0.000	0.000	0.000
	l1=l2=-1/3	0.01	0.135	***0.010***	***0.026***	***0.016***	0.324	0.002	***0.017***	***0.027***	0.839	***-0.019***	***0.016***	***0.030***	0.993	-0.001	0.004	0.005
	(vii)	0.10	0.488	0.004	-0.008	***-0.016***	0.781	***0.014***	***0.015***	***0.011***	0.985	0.004	0.003	0.003	0.999	0.001	0.000	0.000
	Tukey Lambda	0.05	0.335	0.009	-0.006	-0.001	0.598	***-0.023***	***-0.023***	***-0.015***	0.967	0.000	0.003	0.003	0.998	0.001	0.000	0.001
	l1=l2=-2/3	0.01	0.154	***0.025***	***0.038***	***0.029***	0.316	0.002	***0.017***	***0.031***	0.776	***-0.125***	***-0.051***	***-0.029***	0.981	***-0.013***	-0.001	0.004
(d) constant +	(i)	0.10	0.368	-0.002	***-0.011***	***-0.015***	0.630	0.000	0.006	0.002	0.966	0.000	0.001	0.001	1.000	0.000	0.000	0.000
time +	normal	0.05	0.246	0.000	-0.006	***-0.011***	0.503	0.001	***0.011***	0.008	0.935	0.000	0.005	0.004	0.999	0.000	0.000	0.000
unit root		0.01	0.081	0.000	0.000	-0.009	0.258	-0.002	***0.015***	***0.017***	0.817	0.000	***0.031***	***0.032***	0.992	0.000	0.005	0.005
	(ii)	0.10	0.366	-0.004	-0.004	***-0.017***	0.637	-0.004	0.001	0.000	0.965	0.000	0.002	0.002	0.999	0.000	0.000	0.000
	uniform	0.05	0.245	-0.004	***-0.011***	***-0.016***	0.512	-0.008	0.005	0.002	0.932	0.001	0.003	0.006	0.998	0.000	0.000	0.000
		0.01	0.082	-0.005	-0.003	***-0.011***	0.262	***-0.015***	0.005	0.006	0.803	-0.006	***0.023***	***0.026***	0.993	0.000	0.004	0.004
	(iii)	0.10	0.344	***-0.036***	***-0.045***	***-0.049***	0.644	***-0.015***	-0.008	***-0.017***	0.985	0.000	0.001	0.000	1.000	0.000	0.000	0.000
	lognormal	0.05	0.220	***-0.031***	***-0.042***	***-0.042***	0.469	***-0.050***	***-0.030***	***-0.035***	0.952	***-0.011***	-0.004	-0.005	1.000	0.000	0.000	0.000
		0.01	0.084	***-0.017***	***-0.018***	***-0.015***	0.205	***-0.048***	***-0.016***	***-0.013***	0.716	***-0.132***	***-0.041***	***-0.048***	0.992	-0.006	0.000	0.002
	(iv)	0.10	0.369	***-0.054***	***-0.060***	***-0.072***	0.728	0.003	0.003	0.005	0.990	0.006	0.004	0.005	0.998	0.001	0.000	0.000
	Cauchy	0.05	0.228	***-0.054***	***-0.075***	***-0.063***	0.511	***-0.076***	***-0.063***	***-0.066***	0.972	0.001	0.004	0.004	0.997	0.001	0.000	0.001
		0.01	0.120	0.005	-0.007	0.001	0.265	***-0.017***	***0.017***	***0.030***	0.607	***-0.298***	***-0.180***	***-0.166***	0.959	***-0.033***	***-0.022***	***-0.012***
	(v)	0.10	0.372	-0.009	***-0.016***	***-0.022***	0.680	***0.012***	***0.015***	0.009	0.978	0.005	0.005	0.006	1.000	0.000	0.000	0.000
	stable	0.05	0.238	***-0.011***	***-0.023***	***-0.023***	0.539	-0.001	0.005	0.005	0.950	0.003	0.007	0.009	0.999	0.000	0.000	0.000
	a=1.6	0.01	0.091	0.001	-0.002	-0.007	0.274	0.003	***0.019***	***0.029***	0.797	***-0.045***	0.005	***0.015***	0.991	-0.003	0.003	0.006
	(vi)	0.10	0.364	-0.007	***-0.016***	***-0.026***	0.680	0.006	***0.010***	0.005	0.974	0.003	0.002	0.003	0.999	0.000	0.000	0.000
	Tukey Lambda	0.05	0.238	***-0.013***	***-0.023***	***-0.022***	0.541	-0.001	0.008	0.006	0.950	0.005	0.007	0.009	0.998	0.000	0.000	0.000
	l1=l2=-1/3	0.01	0.087	0.001	-0.001	-0.008	0.288	0.003	***0.020***	***0.032***	0.815	***-0.017***	***0.019***	***0.034***	0.991	-0.002	0.004	0.006
	(vii)	0.10	0.360	***-0.029***	***-0.038***	***-0.045***	0.711	0.005	0.007	0.004	0.984	0.005	0.004	0.006	0.999	0.000	0.000	0.001
	Tukey Lambda	0.05	0.225	***-0.035***	***-0.050***	***-0.043***	0.537	***-0.034***	***-0.021***	***-0.021***	0.960	0.001	0.003	0.005	0.997	0.000	0.000	0.001
	l1=l2=-2/3	0.01	0.099	0.000	-0.007	-0.006	0.267	***-0.019***	0.007	***0.026***	0.732	***-0.142***	***-0.060***	***-0.049***	0.979	***-0.014***	-0.003	0.004

Note: entries in the columns labelled EDW are the exact powers of the EDW test for the levels indicated.
Entries in the other columns are the differences between the power of the EDW and one of the DW, PDW, or BDW tests.
See text for further explanation.

Table 2.C

Power of EDW, DW, PDW, and BDW tests against AR(1) alternative with coefficient 1.0

			Sample size															
			12				20				50				100			
Design Matrix	Error Distribution	Nominal Level	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW	EDW	EDW-DW	EDW-PDW	EDW-BDW
(a) constant	(i)	0.10	0.882	0.000	0.005	-0.004	0.984	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
only	normal	0.05	0.827	0.002	0.007	0.004	0.973	0.000	0.002	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.673	0.002	***0.026***	***0.030***	0.937	0.002	***0.012***	***0.016***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(ii)	0.10	0.878	-0.002	0.000	-0.007	0.984	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	uniform	0.05	0.810	-0.006	-0.003	***-0.010***	0.972	-0.001	-0.001	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.637	***-0.025***	-0.006	-0.006	0.926	-0.005	0.004	0.004	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(iii)	0.10	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	lognormal	0.05	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(iv)	0.10	0.925	-0.006	0.004	-0.008	0.991	0.001	0.001	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Cauchy	0.05	0.872	-0.008	***0.013***	-0.003	0.980	-0.005	0.001	-0.002	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.769	***0.028***	***0.077***	***0.077***	0.949	-0.002	***0.024***	***0.017***	0.999	-0.001	0.002	0.001	1.000	0.000	0.000	0.000
	(v)	0.10	0.904	0.001	0.004	-0.002	0.990	0.001	0.001	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	stable	0.05	0.847	0.001	0.005	-0.003	0.978	-0.001	0.001	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	a=1.6	0.01	0.714	***0.015***	***0.047***	***0.045***	0.943	-0.001	***0.013***	***0.012***	1.000	0.000	0.001	0.000	1.000	0.000	0.000	0.000
	(vi)	0.10	0.905	0.003	0.006	-0.004	0.988	0.000	0.001	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Tukey Lambda	0.05	0.851	0.006	***0.010***	0.003	0.978	0.000	0.002	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	l1=l2=-1/3	0.01	0.717	***0.019***	***0.051***	***0.058***	0.941	-0.001	***0.012***	***0.011***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(vii)	0.10	0.915	-0.001	0.005	-0.006	0.990	0.000	0.001	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Tukey Lambda	0.05	0.865	-0.001	0.009	-0.002	0.979	-0.002	0.002	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	l1=l2=-2/3	0.01	0.741	***0.024***	***0.058***	***0.068***	0.946	-0.003	***0.015***	***0.014***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
(b) constant +	(i)	0.10	0.865	0.001	0.005	-0.001	0.983	0.000	0.001	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
normal iid	normal	0.05	0.800	0.002	0.004	0.000	0.969	0.000	0.002	0.002	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.630	-0.001	***0.025***	***0.024***	0.923	0.003	***0.017***	***0.017***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(ii)	0.10	0.859	-0.003	0.000	-0.006	0.981	0.000	0.000	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	uniform	0.05	0.784	-0.008	-0.005	-0.007	0.967	-0.001	0.002	0.002	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.598	***-0.024***	-0.005	-0.005	0.913	-0.003	0.005	0.005	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(iii)	0.10	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	lognormal	0.05	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.999	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(iv)	0.10	0.919	***0.011***	***0.012***	0.004	0.989	0.000	0.000	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Cauchy	0.05	0.865	***0.021***	***0.025***	***0.013***	0.976	-0.005	0.001	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.753	***0.058***	***0.103***	***0.099***	0.943	0.000	***0.024***	***0.021***	0.999	-0.001	0.002	0.002	1.000	0.000	0.000	0.000
	(v)	0.10	0.890	0.005	0.007	0.002	0.987	0.000	0.001	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	stable	0.05	0.828	***0.011***	***0.012***	0.009	0.974	-0.001	0.001	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	a=1.6	0.01	0.681	***0.027***	***0.058***	***0.054***	0.933	-0.001	***0.014***	***0.014***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(vi)	0.10	0.891	0.008	***0.011***	0.002	0.986	0.000	0.001	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Tukey Lambda	0.05	0.832	***0.012***	***0.018***	0.008	0.974	-0.001	0.002	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	l1=l2=-1/3	0.01	0.680	***0.026***	***0.053***	***0.053***	0.931	-0.001	***0.013***	***0.015***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(vii)	0.10	0.907	0.010	***0.014***	0.003	0.988	0.000	0.001	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Tukey Lambda	0.05	0.855	***0.020***	***0.024***	***0.014***	0.975	-0.003	0.003	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	l1=l2=-2/3	0.01	0.722	***0.047***	***0.081***	***0.084***	0.938	-0.002	***0.020***	***0.020***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000

Table 2.C (continued)

Design Matrix	Error Distribution	Nominal Level	12 EDW	12 EDW-DW	12 EDW-PDW	12 EDW-BDW	20 EDW	20 EDW-DW	20 EDW-PDW	20 EDW-BDW	50 EDW	50 EDW-DW	50 EDW-PDW	50 EDW-BDW	100 EDW	100 EDW-DW	100 EDW-PDW	100 EDW-BDW
(c) constant +	(i)	0.10	0.688	0.001	0.006	-0.002	0.937	0.000	0.003	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
time	normal	0.05	0.578	0.002	***0.013***	0.004	0.894	0.001	0.004	0.005	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.352	0.003	***0.045***	***0.026***	0.779	0.006	***0.028***	***0.030***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(ii)	0.10	0.670	-0.008	-0.003	***-0.012***	0.935	-0.001	0.001	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	uniform	0.05	0.555	***-0.013***	-0.002	-0.007	0.890	-0.004	0.001	0.002	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.320	***-0.025***	***0.012***	-0.002	0.763	***-0.014***	0.010	0.009	0.999	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(iii)	0.10	0.736	-0.004	-0.002	***-0.012***	0.965	-0.001	-0.001	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	lognormal	0.05	0.600	-0.007	0.000	***-0.013***	0.929	***-0.011***	-0.004	-0.006	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.325	***-0.023***	***0.030***	0.006	0.792	***-0.033***	-0.009	-0.007	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(iv)	0.10	0.785	-0.010	-0.008	***-0.015***	0.973	0.002	0.001	0.003	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Cauchy	0.05	0.674	0.004	***0.013***	-0.002	0.939	***-0.011***	-0.007	-0.008	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.437	***0.072***	***0.152***	***0.112***	0.866	0.003	***0.030***	***0.024***	0.996	-0.003	0.001	0.001	1.000	0.000	0.000	0.000
	(v)	0.10	0.728	0.003	0.008	-0.003	0.953	0.003	0.004	0.005	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	stable	0.05	0.614	0.008	***0.019***	0.005	0.917	0.001	0.006	0.004	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	a=1.6	0.01	0.378	***0.021***	***0.077***	***0.052***	0.813	0.006	***0.024***	***0.027***	0.999	0.000	0.001	0.000	1.000	0.000	0.000	0.000
	(vi)	0.10	0.731	0.005	0.010	-0.002	0.953	0.002	0.002	0.002	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Tukey Lambda	0.05	0.616	0.009	***0.018***	0.008	0.920	0.002	0.004	0.004	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	l1=l2=-1/3	0.01	0.366	***0.018***	***0.070***	***0.046***	0.817	0.002	***0.027***	***0.028***	0.999	0.000	0.000	0.001	1.000	0.000	0.000	0.000
	(vii)	0.10	0.769	0.003	0.007	-0.003	0.967	0.003	0.002	0.004	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Tukey Lambda	0.05	0.648	0.009	***0.016***	-0.001	0.933	-0.004	-0.003	-0.003	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	l1=l2=-2/3	0.01	0.400	***0.049***	***0.113***	***0.080***	0.845	0.000	***0.023***	***0.026***	0.998	-0.001	0.000	0.000	1.000	0.000	0.000	0.000
(d) constant +	(i)	0.10	0.567	-0.002	-0.007	***-0.014***	0.911	0.000	0.005	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
time +	normal	0.05	0.441	0.001	-0.004	-0.006	0.857	0.001	0.006	0.002	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
unit root		0.01	0.212	0.000	0.008	0.002	0.709	-0.001	***0.029***	***0.026***	0.999	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(ii)	0.10	0.561	-0.003	-0.006	***-0.016***	0.911	-0.001	0.001	0.002	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	uniform	0.05	0.436	-0.003	-0.007	***-0.011***	0.857	-0.002	0.003	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.196	***-0.011***	0.000	***-0.011***	0.708	***-0.013***	***0.014***	***0.017***	0.999	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(iii)	0.10	0.558	***-0.036***	***-0.042***	***-0.055***	0.941	-0.004	0.000	-0.005	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	lognormal	0.05	0.392	***-0.046***	***-0.051***	***-0.058***	0.885	***-0.016***	-0.010	***-0.014***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.152	***-0.028***	***-0.019***	***-0.039***	0.710	***-0.048***	***-0.019***	***-0.019***	0.999	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	(iv)	0.10	0.579	***-0.048***	***-0.056***	***-0.076***	0.954	0.001	0.002	0.001	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Cauchy	0.05	0.402	***-0.078***	***-0.079***	***-0.084***	0.902	***-0.021***	***-0.016***	***-0.018***	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
		0.01	0.165	0.008	0.010	***-0.021***	0.790	***-0.011***	***0.027***	***0.016***	0.995	-0.004	0.001	-0.001	1.000	0.000	0.000	0.000
	(v)	0.10	0.588	-0.010	***-0.013***	***-0.026***	0.931	0.002	0.006	0.005	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	stable	0.05	0.436	***-0.019***	***-0.020***	***-0.030***	0.881	0.000	0.004	0.005	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	a=1.6	0.01	0.190	0.000	0.003	-0.007	0.745	0.002	***0.031***	***0.028***	0.999	0.000	0.001	0.000	1.000	0.000	0.000	0.000
	(vi)	0.10	0.583	-0.006	***-0.013***	***-0.026***	0.931	0.001	0.005	0.003	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Tukey Lambda	0.05	0.432	***-0.015***	***-0.019***	***-0.030***	0.888	0.000	0.007	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	l1=l2=-1/3	0.01	0.189	0.003	0.010	-0.007	0.759	0.001	***0.036***	***0.034***	0.999	0.000	0.000	0.001	1.000	0.000	0.000	0.000
	(vii)	0.10	0.581	***-0.029***	***-0.033***	***-0.051***	0.948	0.002	0.003	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	Tukey Lambda	0.05	0.416	***-0.051***	***-0.052***	***-0.058***	0.897	***-0.012***	-0.007	-0.010	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
	l1=l2=-2/3	0.01	0.167	0.000	0.003	***-0.017***	0.774	***-0.013***	***0.023***	***0.020***	0.998	-0.001	0.000	0.000	1.000	0.000	0.000	0.000

Note: entries in the columns labelled EDW are the exact powers of the EDW test for the levels indicated.
Entries in the other columns are the differences between the power of the EDW and one of the DW, PDW, or BDW tests.
See text for further explanation.

Table 3
Power loss of the PDW versus the EDW.

Serial Correlation Coefficient	Sample Size	Resamples 199	999
0.5	12	0.014	0.004
	20	0.031	0.013
	50	0.023	0.003
1	12	0.026	0.006
	20	0.012	0.004
	50	0.000	0.000

Note: For design (a) and normal errors (i) the entry gives the exact power difference between the EDW and PDW.

Table 4
Size and Power of the BDW versus the PDW

	Sample Size	Test	Resamples 199	999
Level	50	PDW	0.103	0.103
		BDW	0.080	0.081
	100	PDW	0.104	0.103
		BDW	0.072	0.070
Power against 0.1 alternative	50	PDW	0.426	0.434
		BDW	0.219	0.197
	100	PDW	0.673	0.686
		BDW	0.524	0.547

Note: For design (a) and normal errors (i) the entry gives the exact level or power for the indicated tests at the indicated resampling size.